MACM 101, Fall 2004, SFU Surrey, Sections D2 and D3

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Practice exercises for Midterm #1

The questions on the actual test are not necessarily similar to these ones, nor is the length of the actual test similar to this one. These exercises will not be graded; the points in each question (between curly brackets) are provided just to illustrate how the exercises would be graded if they were part of an exam.

Part A: Short Answer

In each of the following questions, place a single answer in the corresponding blank. Do not justify your answer. For true/false answers, please write the entire word true or false to avoid ambiguity.

1. {1} The Duality Principle for Propositional Logic states that, if a formula A is logically equivalent to a formula B and if the formula A' is ————————————- and the formula B' is ———————————— then ————————————————.
2. {1} One idempotence property for propositions states that $P \bigwedge P \Leftrightarrow$ ————-
3. {1} For any two sets A and B, if $A \subset B$, then $A \cup B =$ ————————————-
4. {1} If little Jack Horner sat in the corner, then he ate his Thanksgiving pie. Little Jack Horner did not sit in the corner. Therefore, he did not eat his Thanksgiving pie. (Is it a valid or invalid argument ?) ————————————

Let $A = \{2, x, y, z\}, B = \{y, 2, \emptyset\}$, and $C = \{Sue, 1, \{x, y\}, 2\}$. Questions 5 to 10 below refer to these three sets. For true/false answers, please write the entire word *true* or *false* to avoid ambiguity.

5 {1} $|P(B)| =$ ————————————

6 {1} $B \subseteq C =$ ————————————

7 {1} $B \cap C$ ————————————-

8 {1} $B - A$ ————————————

9 {1} $|A \cap B \cup P(A)| =$ ————————————

10 {1} $\emptyset \in P(B)$ (true/false) ————————————

Part B: Long Answer

1. {6} Prove using the definitions of set relations and operations (referred to as "element argument" type of proofs in the textbook):
   For all sets A, B, and C, $(A - B) \cup (A \cap B) \subseteq A$

2. {8} Prove that the following derivations are valid. Perform one step at a time and give a reason for each step.

   (a) $(\neg p \bigvee q) \rightarrow r$
   $r \rightarrow (s \bigvee t)$
   $\neg s \bigwedge \neg u$
   $\neg u \rightarrow \neg t$
   ---
   p

   (b) $[p \bigwedge (p \rightarrow q) \bigwedge (s \bigvee r) \bigwedge (r \rightarrow \neg q)] \Rightarrow s$

   (c) $p \bigvee q$
   $\neg p \bigvee r$
   $\neg r$
   ---
   q

3. {4} Prove that the following statement is a tautology. Use logical equivalences, such as those referred to in the textbook as "the laws of logic". Perform one step at a time and name the law (logical equivalence) used at each step. $(p \rightarrow q \bigwedge \neg q) \rightarrow \neg p$

4. {3} Use truth tables to prove the following logical equivalence: $p \bigvee (p \bigwedge q) \Leftrightarrow p$

5. {2} Let $A = \{2, 3, 5, \{1, 2\}, s, (2, 3), \emptyset\}$. Find $|A|$.

6. {4} Use element arguments to prove the following set properties:

   (a) $\overline{A \cup B} = \bar{A} \cap \bar{B}$
   (b) $A - B = A \cap \bar{B}$

7. {4} Let the domain be the integer numbers and let the following predicates stand for the following equations and inequations:
   $p(x)$: $x \geq 0$
   $q(x)$: $x^2 \geq 0$
   $r(x)$: $x^2 - 3x - 4 = 0$
   $s(x)$: $x^2 - 5 \geq 0$

   Find the truth value of the following. You must justify your answer.

   (a) $\exists x.[p(x) \bigwedge r(x)]$

(b) $\forall x.[p(x) \rightarrow q(x)]$

(c) $\forall x.[q(x) \rightarrow s(x)]$

8. {4} Write the negation of each of the following true statements. (It is recommended to first express the original statements symbolically, then negate the formula, then simplify and finally translate back to English).

   (a) For all integers n if n is not divisible by 2 then n is odd.

   (b) If k, m, n are integers where $k - m$ and $m - n$ are odd, then $k - n$ is even.

9. {7} *The potential murderer*
   This is the evidence we have. Everybody is a suspect. Mr. Jones is the butler. Mrs Flower and Mr. Nature are gardeners. No-one can be both a butler and a gardener. A gardener whose garden is ruined becomes very upset. Mr. Jones ruined Mrs Flower's garden. Anyone who is a very upset suspect is a potential murderer. Is there a potential murderer? who is it??
   Justify your answer using a derivation in predicate logic.

   The following predicates should be used:
   $b(x)$: x is a butler.
   $g(x)$: x is a gardener.
   $r(x)$: x's graden was ruined.
   $u(x)$: x is very upset.
   $p(x)$: x is a potential murderer.

   The following constants should be used:
   j: Mr. Jones
   f: Mrs. Flower
   n: Mr. Nature