

MACM 101-D3, Fall 2004, SFU Surrey

Midterm #2 Preparation.

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Welcome to the second MACM-101 midterm.

- The duration of the midterm is 80 minutes.
 - You are not allowed access to any materials with the exception of your crib-sheet.
 - For each of the exercises it is more important that you explain and justify your calculations, than the actual result.
 - In absence of justifications, a numeric result, even if correct, may be disconsidered entirely.
 - When you arrive at a formula that contains just numbers (e.g. $\frac{5! \cdot 20!}{4 \cdot 3^{11}}$), it is not necessary to calculate the final numeric result.
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1. Expand the following:

(a) {5p} $\sum_{k=2}^6 k^2$

(b) {5p} $\sum_{k=0}^4 P(5, k)$

2. Expand and calculate the following:

(a) {5p} $\prod_{k=2}^6 \frac{\binom{k}{k-2}}{\binom{k+1}{k-1}}$

(b) {5p} $\sum_{k=0}^n \frac{P(n, k)}{P(k, k)}$

3. {10p} Consider A, B, C finite sets. Prove the following property (distributivity of Cartesian product over union):

$$(A \cup B) \times C = (A \times C) \cup (B \times C)$$

4. {10p} Show that $\forall n, k$, where $k, n \in \mathbb{N}^*$ and $k < n - 1$:

$$\binom{n}{k} - \binom{n}{k+2} = \binom{n+1}{k+1} - \binom{n+1}{k+2}$$

5. A US zip code is made of 5 digits (e.g. 12345). A Canadian postal code is made of six symbols, alternating letters and digits (e.g. V5A 1S6). You are asked below to calculate how many such codes there are, based on increasing restrictions.

Some steps are based on steps before them. You can skip steps which you don't know.

- (a) {5p} How many possible US zip codes (as above) are there? (N_1)
 - (b) {5p} No¹ US zip code can start with '0'. How many US zip codes are left? (N_2)
 - (c) {5p} How many possible Canadian postal codes (as above) are there? (N_3)
 - (d) {5p} Canadian postal codes can start with only one of 13 letters indicating the province or territory (e.g. 'V' for British Columbia). How many Canadian postal codes are left? (N_4)
 - (e) {5p} How many strings in total are there that can represent US zip codes and Canadian postal codes (following all the above restrictions)? (N_5)
6. {10p} "Crazy Mike's" pizza place is making a new kind of pizza: the "Sausage Extreme". Each such pizza is covered with exactly 16 slices of 3 types of sausage: "Italian", "Turkey" and "Bologna".

If you order 4 "Sausage Extreme" pizzas, and you find that each has more than 10 slices of sausage of the same type, you win a trip to Buffalo, New York.

Calculate the proportion of winning orders of 4 pizzas, compared to all possible orders of 4 pizzas. Explain.

¹This is probably not true in real life.