SFU MACM-101-D3 2004-2 week 5

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Set operations

Consider A, B sets.

- union : $A \cup B = \{x \quad | \quad x \in A \vee x \in B\}$
- intersection: $A \cap B = \{x \quad | \quad x \in A \wedge x \in B\}$
- difference: $A - B = \{x \quad | \quad x \in A \wedge x \notin B\}$
- symmetric difference: $A \Delta B = \{x \quad | \quad x \in A \cup B \wedge x \notin A \cap B\}$

Complement

Definition 1 *Consider a set A from a universe U,* the complement of A *(written $\overline{A}$) is:*

$$\overline{A} = U - A = \{x \quad | \quad x \notin A\}$$

Observation 1 *The concept of universe is necessary for defining the complement of a set.*

Suggestion 1 *If unsure, use a set that* includes at least all sets that are necessary for a given problem.

Laws of set theory

Observation 2 *The laws of set theory are very similar (parallel) to the laws of logic.*

Sets of Numbers

- $\mathbb{N} = \{0, 1, 2, 3, \ldots\}$: *natural numbers*
- $\mathbb{Z} = \{n, -n \quad | \quad n \in \mathbb{N}\}$: *integer numbers*
- $\mathbb{Q} = \{\frac{a}{b} \quad | a, b \in \mathbb{Z} \wedge b \neq 0\}$: *rational numbers*
- $\mathbb{R}$: real numbers (defined as the continuous closure of $\mathbb{Q}$)
- $\mathbb{C} = \{x + iy \quad | \quad x, y \in \mathbb{R} \wedge i = \sqrt{-1}\}$: *complex numbers* (closed on root functions)

Sets of Numbers

Observation 3 *The set of natural numbers ($\mathbb{N}$) is usually defined as the set of cardinalities of all finite sets.*

Observation 4

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R} \subset \mathbb{C}$$

Sets of Numbers

Observation 5

$$\mathbb{R} \neq \mathbb{Q}$$

Observation 6

$$\mathbb{R} \neq \mathbb{Q} \cup \{ \sqrt[x]{y} \quad | \quad x, y \in \mathbb{Z} \wedge x \neq 0 \}$$

Integer sequences, intervals

Definition 2 (Interval) *Considering $a, b \in \mathbb{R} \wedge a < b$:*

- $[a, b] = \{x \quad | \quad a \leq x \leq b\}$
- $(a, b] = \{x \quad | \quad a < x \leq b\}$
- $[a, b) = \{x \quad | \quad a \leq x < b\}$
- $(a, b) = \{x \quad | \quad a < x < b\}$

Definition 3 (Integer sequence) *Considering $a, b \in \mathbb{Z} \wedge a < b$, the a to b sequence (written $a..b$) is:*

$$a..b = [a, b] \cap \mathbb{Z}$$

Cardinality and set operations

Observation 7 *Considering A, B finite sets:*

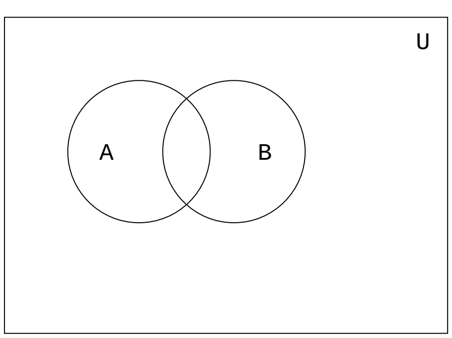
- $|A \cup B| \leq |A| + |B|$
- $|A \cap B| \leq |A| \wedge |A \cap B| \leq |B|$
- $|A - B| \leq |A|$
- $|A \Delta B| \leq |A \cup B|$

Sets and Logic

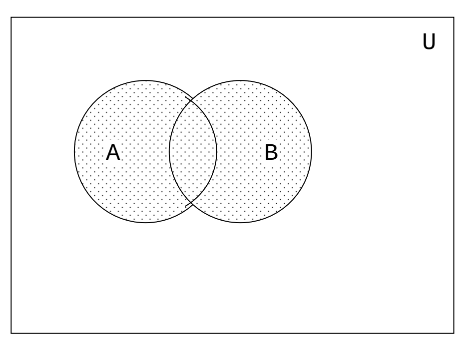
Observation 8 *Logic connectives and set operations are deeply interrelated, as follows:*

- *union vs. disjunction ($\cup$ vs. $\vee$)*
- *intersection vs. conjunction ($\cap$ vs. $\wedge$)*
- *symmetric difference vs. exclusive or (Δ vs. $\veebar$)*
- *inclusion vs. implication ($\subseteq$ vs. $\rightarrow$)*
- *set equality vs. biconditional ($=$ vs. $\leftrightarrow$)*
- *complement vs. negation ($\overline{A}$ vs. $\neg$)*

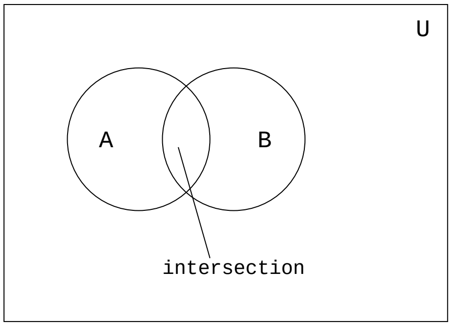
Venn diagrams



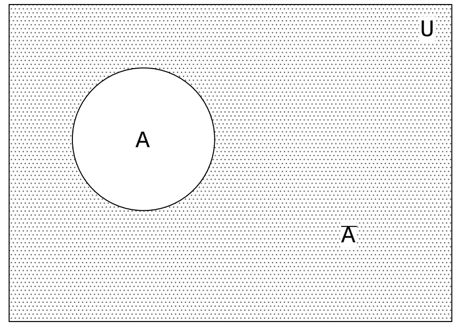
Venn diagrams: union



Venn diagrams: intersection



Venn diagrams: complement



Set operations and truth tables

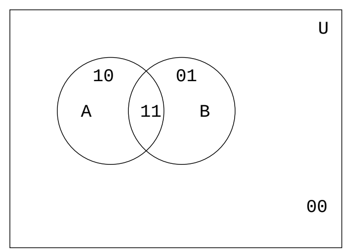
Observation 9 *Element-in-set truth tables can be used in set-theory proofs.*

Example (the statements are "$x \in$ the heading set"):

A	B	$A \cup B$	$A \cap B$	$A - B$	$B - A$	$(A \cup B) - (B \cap A)$	$(A - B) \cup (B - A)$
0	0	0	0	0	0	0	0
0	1	1	0	0	1	1	1
1	0	1	0	1	0	1	1
1	1	1	1	0	0	0	0

Set truth tables and Venn diagrams

Observation 10 *All areas in a Venn diagram can be marked with a truth code.*



More about cardinality and set operations

Observation 11 *Considering A, B finite sets:*

- $|A \cup B| = |A| + |B| - |A \cap B|$
- $|A - B| + |A \cap B| = |A|$
- $|A \Delta B| = |A \cup B| - |A \cap B|$

Power set

Definition 4 (Power set) *Considering a set A, the power set of A is a set containing all subsets of A:*

$$\mathcal{P}(A) = \{X \quad | \quad X \subseteq A\}$$

Observation 12

$$|\mathcal{P}(A)| = 2^{|A|}$$