SFU MACM-101-D3 2004-2 week 6

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October 28, 2004

Revision : 1.4

Binomial Theorem

Theorem 1 (Binomial theorem) *For all* $n \in \mathbb{N}$*:*

$$\begin{aligned}(x+y)^n &= \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k} \\ &= \binom{n}{0} x^0 y^n + \binom{n}{1} x^1 y^{n-1} + \ldots + \binom{n}{n} x^n y^0\end{aligned}$$

Proof: requires induction.

Properties of combinations

Observation 1 *For all* $n \in \mathbb{N}$*:*

$$\sum_{k=0}^{n} \binom{n}{k} = 2^n$$

Observation 2 *For all* $n, k \in \mathbb{N}$ *for which* $n \geq k$*:*

$$\binom{n}{k} = \binom{n}{n-k}$$

Observation 3 *For all* $n, k \in \mathbb{N}$ *for which* $k > 0$ *and* $n > k$*:*

$$\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$$

Pascal's triangle

$$\binom{0}{0}$$

$$\binom{1}{0} \quad \binom{1}{1}$$

$$\binom{2}{0} \quad \binom{2}{1} \quad \binom{2}{2}$$

$$\binom{3}{0} \quad \binom{3}{1} \quad \binom{3}{2} \quad \binom{3}{3}$$

$$\binom{4}{0} \quad \binom{4}{1} \quad \binom{4}{2} \quad \binom{4}{3} \quad \binom{4}{4}$$

etc

Pascal's triangle

```
        1
      1   1
    1   2   1
  1   3   3   1
1   4   6   4   1
1  5 10 10  5  1
.......etc.........
```

Multisets

Note 1 (Multiset) *A* multiset *is a well defined collection of objects, irrelevant of order, but where the same object can appear zero or more times.*

Examples:

- $\{1, 1, 2, 3, 2, 1, 2, 2\}$
- $\{a, aa, aa, aa\}$

Link:

http://mathworld.wolfram.com/Multiset.html

Multisets

Observation 4 *In a multiset, the order of elements does not count.*

Note 2 *The set of objects that appear in a multiset is called here the* base set *of the multiset.*

Example:

$\{1, 1, 2, 2, 2\}$ is a multiset with the base set $\{1, 2, 3\}$.

Note 3 *The cardinality ($|M|$) of a multiset M is the total number of occurrences of all elements in the multiset.*

Example: $|\{1, 1, 2, 3\}| = 4$

Combinations with repetition

Definition 1 (Combinations with repetitions) *The number of all different multisets of cardinality r using a base set of size n is called* Combinations with repetition of n elements taken r at a time.

Observation 5 *The formula for combinations with repetition of n elements taken r at a time:*

$$\binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$