

SFU MACM-101-D3 2004-2 week 6

Manuel Zahariev  
E-mail: manuelz@cs.sfu.ca

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Binomial Theorem

**Theorem 1 (Binomial theorem)** For all  $n \in \mathbb{N}$ :

$$(x + y)^n = \sum_{k=0}^n \binom{n}{k} x^k y^{n-k}$$

$$= \binom{n}{0} x^0 y^n + \binom{n}{1} x^1 y^{n-1} + \dots + \binom{n}{n} x^n y^0$$

Proof: requires induction.

Properties of combinations

**Observation 1** For all  $n \in \mathbb{N}$ :

$$\sum_{k=0}^n \binom{n}{k} = 2^n$$

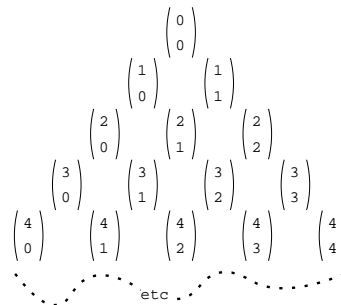
**Observation 2** For all  $n, k \in \mathbb{N}$  for which  $n \geq k$ :

$$\binom{n}{k} = \binom{n}{n-k}$$

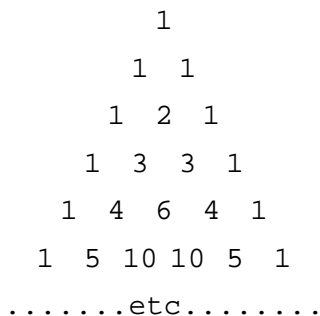
**Observation 3** For all  $n, k \in \mathbb{N}$  for which  $k > 0$  and  $n > k$ :

$$\binom{n}{k-1} + \binom{n}{k} = \binom{n+1}{k}$$

Pascal's triangle



Pascal's triangle



Multisets

**Note 1 (Multiset)** A **multiset** is a well defined collection of objects, irrelevant of order, but where the same object can **appear zero or more times**.

Examples:

- {1, 1, 2, 3, 2, 1, 2, 2}
- {a, aa, aa, aa}

Link:

<http://mathworld.wolfram.com/Multiset.html>

## Multisets

**Observation 4** In a multiset, the *order of elements does not count*.

**Note 2** The set of objects that appear in a multiset is called here the *base set* of the multiset.

Example:

$\{1, 1, 2, 2, 2\}$  is a multiset with the base set  $\{1, 2, 3\}$ .

**Note 3** The cardinality ( $|M|$ ) of a multiset  $M$  is the total number of occurrences of all elements in the multiset.

Example:  $|\{1, 1, 2, 3\}| = 4$

## Combinations with repetition

**Definition 1 (Combinations with repetitions)** The number of all different multisets of cardinality  $r$  using a base set of size  $n$  is called *Combinations with repetition of  $n$  elements taken  $r$  at a time*.

**Observation 5** The formula for combinations with repetition of  $n$  elements taken  $r$  at a time:

$$\binom{n+r-1}{r} = \frac{(n+r-1)!}{r!(n-1)!}$$