CMPT 379
Compilers

Anoop Sarkar

http://www.cs.sfu.ca/~anoop

Parsing - Roadmap

- Parser:
  - decision procedure: builds a parse tree
- Top-down vs. bottom-up
- LL(1) – Deterministic Parsing
  - recursive-descent
  - table-driven
- LR(k) – Deterministic Parsing
  - LR(0), SLR(1), LR(1), LALR(1)
- Parsing arbitrary CFGs – Polynomial time parsing

Top-Down vs. Bottom Up

Grammar: S → A B
A → c | ε
B → cbB | ca

Input String: ccbca

Top-Down/leftmost		Bottom-Up/rightmost	
S ⇒ AB	S→AB	ccbca ⇐ Acbca	A→c
⇒ cB	A→c	⇐ AcbB	B→ca
⇒ ccbB	B→cbB	⇐ AB	B→cbB
⇒ ccbca	B→ca	⇐ S	S→AB

Top-Down: Backtracking

S → A B

A → c | ε

B → cbB | ca

True/False

S ⇒* cbca?

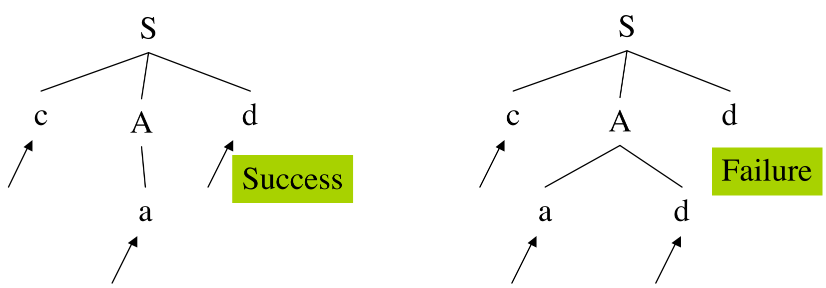
S	cbca	try S→AB
AB	cbca	try A→c
cB	cbca	match c
B	bca	dead-end, try A→ε
εB	cbca	try B→cbB
cbB	cbca	match c
bB	bca	match b
B	ca	try B→cbB
cbB	ca	match c
bB	a	dead-end, try B→ca
ca	ca	match c
a	a	match a, Done!

Backtracking

S → cAd | c
A → a | ad

Input: cad

S → cAd | c
A → ad | a



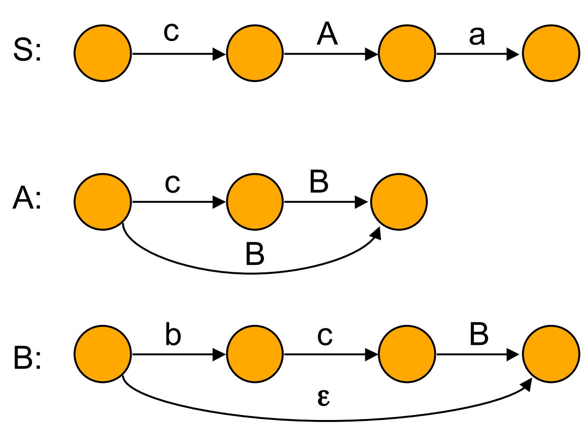
For some grammars, rule ordering is crucial for backtracking parsers, e.g $S \rightarrow aSa$, $S \rightarrow aa$

Transition Diagram

$S \rightarrow cAa$

$A \rightarrow cB \mid B$

$B \rightarrow bcB \mid \varepsilon$



Predictive Top-Down Parser

- Knows which production to choose based on single lookahead symbol
- Need LL(1) grammars
  - First L: reads input Left to right
  - Second L: produce Leftmost derivation
  - 1: one symbol of lookahead
- Can't have left-recursion
- Must be left-factored (no left-factors)
- Not all grammars can be made LL(1)

Leftmost derivation for **id + id * id**

E → E + E	E ⇒ E + E
E → E * E	⇒ **id** + E
E → (E)	⇒ **id** + E * E
E → - E	⇒ **id** + **id** * E
E → **id**	⇒ **id** + **id** * **id**

$E \Rightarrow^{*}_{lm}$ id + E * E

Predictive Parsing Table

Productions	
1	**T → F T’**
2	**T’ → ε**
3	**T’ → * F T’**
4	**F → id**
5	**F → (T)**

	*	(	)	id	$
T		**T → F T’**		**T → F T’**	
T’	**T’ → * F T’**		**T’ → ε**		**T’ → ε**
F		**F → (T)**		**F → id**	

Trace “(id)*id”

	*	(	)	id	$
T		**T → FT’**		**T → FT’**	
T’	**T’ → *FT’**		**T’ → ε**		**T’ → ε**
F		**F → (T)**		**F → id**	

Stack	Input	Output
$T	(id)*id$	
$T’F	(id)*id$	**T → F T’**
$T’)T(	(id)*id$	**F → (T)**
$T’)T	id)*id$	
$T’)T’F	id)*id$	**T → F T’**
$T’)T’id	id)*id$	**F → id**
$T’)T’	)*id$	
$T’)	)*id$	**T’ → ε**

Trace “(id)*id”

	*	(	)	id	$
T		**T → FT’**		**T → FT’**	
T’	**T’ → *FT’**		**T’ → ε**		**T’ → ε**
F		**F → (T)**		**F → id**	

Stack	Input	Output
$T’	*id$	
$T’F*	*id$	**T’ → * F T’**
$T’F	id$	
$T’id	id$	**F → id**
$T’	$	
$	$	**T’ → ε**

Table-Driven Parsing

stack.push($); stack.push(S);
a = input.read();
forever do begin
  X = stack.peek();
  if X = a **and** a = $ **then** return SUCCESS;
  elsif X = a **and** a != $ **then**
    pop X; a = input.read();
  elsif X != a **and** X ∈ **N and** M[X,a] **then**
    pop X; push right-hand side of M[X,a];
  else ERROR!
end

Predictive Parsing table

- Given a grammar produce the predictive parsing table
- We need to to know for all rules $A \rightarrow \alpha \mid \beta$ the lookahead symbol
- Based on the lookahead symbol the table can be used to pick which rule to push onto the stack
- This can be done using two sets: FIRST and FOLLOW

FIRST and FOLLOW

$$a \in \mathrm{FIRST}(\alpha) \text{ if } \alpha \Rightarrow^* a\beta$$

$$\text{if } \alpha \Rightarrow^* \epsilon \text{ then } \epsilon \in \mathrm{FIRST}(\alpha)$$

$$a \in \mathrm{FOLLOW}(A) \text{ if } S \Rightarrow^* \alpha A a \beta$$

$$a \in \mathrm{FOLLOW}(A) \text{ if } S \Rightarrow^* \alpha A \gamma a \beta$$

$$\text{and } \gamma \Rightarrow^* \epsilon$$

Conditions for LL(1)

- Necessary conditions:
  - no ambiguity
  - no left recursion
  - Left factored grammar
- A grammar G is LL(1) iff - whenever
  $A \rightarrow \alpha \mid \beta$

1. $First(\alpha) \cap First(\beta) = \emptyset$
2. $\alpha \Rightarrow^* \varepsilon$ implies $!(\beta \Rightarrow^* \varepsilon)$
3. $\alpha \Rightarrow^* \varepsilon$ implies $First(\beta) \cap Follow(A) = \emptyset$

ComputeFirst(α: string of symbols)

// assume $\alpha = X_1\ X_2\ X_3 \ldots X_n$
if $X_1 \in$ **T then** First[α] := $\{X_1\}$
else begin
  i:=1; First[α] := ComputeFirst(X_1)\\{**ε**};
  while $X_i \Rightarrow^*$ **ε do begin**
    if i < n **then**
      First[α] := First[α] ∪ ComputeFirst(X_{i+1})\\{**ε**};
    else
      First[α] := First[α] ∪ {**ε**};
    i := i + 1;
  end
end

ComputeFirst(α: string of symbols)

```
// assume α = X_1 X_2 X_3 … X_n
if X_1 ∈ T then First[α] := {X_1}
else begin
  i:=1; First[α] := ComputeFirst(X_1)\{ε};
  while X_i ⇒* ε do begin
    if i < n then
      First[α] := First[α] ∪ ComputeFirst(X_{i+1})\{ε};
    else
      First[α] := First[α] ∪ {ε}; break;
  i := i + 1;
  end
end
```

Recursion in computing FIRST causes problems when faced with left-recursive grammars

ComputeFirst; modified

```
foreach X ∈ T do First[X] := X;
foreach p ∈ P : X → ε do First[X] := {ε};
repeat foreach X ∈ N, p : X → Y_1 Y_2 Y_3 … Y_n do
  begin i:=1;
    while Y_i ⇒* ε and i <= n do begin
      First[X] := First[X] ∪ First[Y_i]\{ε};
      i := i+1;
    end
    if i = n+1 then First[X] := First[X] ∪ {ε};
    else First[X] := First[X] ∪ First[Y_i];
until no change in First[X] for any X;
```

ComputeFirst; modified

foreach X ∈ **T do** First[X] := X;

foreach p ∈ **P** : X → **ε do** First[X] := {**ε**};

repeat foreach X ∈ **N**, p : X → Y_1 Y_2 Y_3 … Y_n **do**

  begin i:=1;

   while Y_i ⇒*

    First[X] :=

    i := i+1;

   end

  if i = n+1 **then** First[X] := First[X] ∪ {**ε**};

  else First[X] := First[X] ∪ First[Y_i];

until no change in First[X] for any X;

> Non-recursive FIRST computation
> works with left-recursive grammars.
> Computes a fixed point for FIRST[X]
> for all non-terminals X in the grammar.
> But this algorithm is very inefficient.

ComputeFollow

Follow(S) := {$};

repeat

 foreach p ∈ **P do**

  case p = A → αBβ **begin**

   Follow[B] := Follow[B] ∪ ComputeFirst(β)\{**ε**};

   if ε ∈ First(β) **then**

    Follow[B] := Follow[B] ∪ Follow[A];

  end

  case p = A → αB

   Follow[B] := Follow[B] ∪ Follow[A];

until no change in any Follow[N]

Example First/Follow

$S \rightarrow AB$

$A \rightarrow c \mid \varepsilon$ Not an LL(1) grammar

$B \rightarrow cbB \mid ca$

First(A) = {c, ε}

First(B) = {c}

First(cbB) =

First(ca) = {c}

First(S) = {c}

Follow(A) = {c}

Follow(A) ∩

First(c) = {c}

Follow(B) = {$}

Follow(S) = {$}

ComputeFirst on Left-recursive Grammars

- ComputeFirst as defined earlier loops on left-recursive grammars
- Here is an alternative algorithm for ComputeFirst
  1. Compute non left-recursive cases of FIRST
  2. Create a graph of recursive cases where FIRST of a non-terminal depends on another non-terminal
  3. Compute Strongly Connected Components (SCC)
  4. Compute FIRST starting from root of SCC to avoid cycles
- Unlike top-down LL parsing, bottom-up LR parsing allows left-recursive grammars, so this algorithm is useful for LR parsing

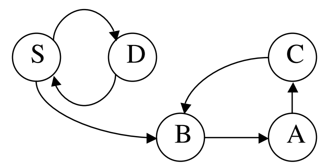
ComputeFirst on Left-recursive Grammars

- S → BD | D
- D → d | Sd

- A → CB | a
- C → Bb | ε
- B → Ab | b

Compute Strongly Connected Components

$FIRST_0[A] := \{a, b\}$
$FIRST_0[C] := \{\}$
$FIRST_0[B] := \{b\}$
$FIRST_0[S] := \{b, d\}$
$FIRST_0[D] := \{d\}$



2 SCCs: e.g. consider B-A-C

$FIRST[B] := FIRST_0[B] + FIRST[A]$

$FIRST[A] := FIRST_0[A] + FIRST[C]$

$FIRST[C] := FIRST_0[C] + FIRST_0[B]$

$FIRST[C] := FIRST[C] + \{\varepsilon\}$

Converting to LL(1)

S → AB
A → c | ε
B → cbB | ca

Note that grammar is regular: c? (cb)* ca

c (c b c b … c b) c a
(c b c b … c b) c a

→

c c (b c b … c b c) a
c (b c b … c b c) a

same as:
c c? (bc)* a

S → cAa
A → cB | B
B → bcB | ε

Verifying LL(1) using F/F sets

$S \rightarrow cAa$

$A \rightarrow cB \mid B$

$B \rightarrow bcB \mid \varepsilon$

First(A) = {b, c, ε}
Follow(A) = {a}

First(B) = {b, ε}
Follow(B) = {a}

First(S) = {c}
Follow(S) = {$}

Building the Parse Table

- Compute First and Follow sets
- For each production $A \rightarrow \alpha$
  - foreach $a \in$ First(α) add $A \rightarrow \alpha$ to M[A,a]
  - If **ε** $\in$ First(α) add $A \rightarrow \alpha$ to M[A,b] for each b in Follow(A)
  - If **ε** $\in$ First(α) add $A \rightarrow \alpha$ to M[A,$] if $ $\in$ Follow(α)
  - All undefined entries are errors

Revisit conditions for LL(1)

- A grammar G is LL(1) iff - whenever $A \rightarrow \alpha \mid \beta$
  1. $First(\alpha) \cap First(\beta) = \emptyset$
  2. $\alpha \Rightarrow^* \varepsilon$ implies $!(\beta \Rightarrow^* \varepsilon)$
  3. $\alpha \Rightarrow^* \varepsilon$ implies $First(\beta) \cap Follow(A) = \emptyset$
- No more than one entry per table field

Error Handling

- Reporting & Recovery
  - Report as soon as possible
  - Suitable error messages
  - Resume after error
  - Avoid cascading errors
- Phrase-level vs. Panic-mode recovery

Panic-Mode Recovery

- Skip tokens until *synchronizing set* is seen
  - Follow(A)
    - garbage or missing things after
  - Higher-level start symbols
  - First(A)
    - garbage before
  - Epsilon
    - if nullable
  - Pop/Insert terminal
    - "auto-insert"
- Add "synch" actions to table

Summary so far

- LL(1) grammars, necessary conditions
    - No left recursion
    - Left-factored
- Not all languages can be generated by LL(1) grammar
- LL(1) – Parsing: $O(n)$ time complexity
  - recursive-descent and table-driven predictive parsing
- LL(1) grammars can be parsed by simple predictive recursive-descent parser
  - Alternative: table-driven top-down parser