CMPT-379 Compilers

Anoop Sarkar
http://www.cs.sfu.ca/~anoop

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Programming Languages and Formal Language Theory

- We ask the question: *Does a particular formal language describe some key aspect of a programming language*
- Then we find out if that language **isn't** in a particular language class

Programming Languages and Formal Language Theory

- For example, if we abstract some aspect of the programming language structure to the formal language: $\{ww^R \mid \text{ where } w \in \{a, b\}^*, w^R \text{ is the reverse of } w\}$ we can then ask if this language is a regular language
- If this is false, i.e. the language is not regular, then we have to go beyond regular languages

Recursion in Regular Languages

- Consider a regular expression for arithmetic expressions:
  $2 + 3 * 4$
  $8 * 10 + -24$
  $2 + 3 * -2 + 8 + 10$
- `\s*-?\s*\d+\s*((\+|\*)\s*-?\s*\d+\s*)*`
- Can we compute the *meaning* of these expressions?

Recursion in Regular Languages

- Construct the finite state automata and associate the meaning with the state sequence
- However, this solution is missing something crucial about arithmetic expressions – *what is it?*

Do Programming Languages belong to Regular Languages

- Consider the following arithmetic expressions
  - $(((2) + (3)) * (4))$
  - $((8) * ((10) + (-24)))$
- Map $(\rightarrow a$ and $) \rightarrow b$. Map everything else to ϵ (keep only the tree structure)
- This results in strings like *aaababbabb* and *aabaababbb*
- What is a good description of this language?
- Let's call it $L = \{a^n b^n : n \geq 0\}$ or simply $a^n b^n$ for short.

Pumping Lemma proofs

- Is L a regular language?
- For any infinite set of strings generated by a finite-state machine if you consider a string that is long enough from this set, there has to be a loop which visits the same state at least twice (from *the pigeonhole principle*)
- Thus, in a regular language L, there are strings x, y, z such that $xy^iz \in L$ for $i \geq 0$ where $y \neq \epsilon$
- We can use this basic characteristic of regular languages to show that $a^n b^n$ cannot be regular

The Chomsky Hierarchy

- **unrestricted** or **type-0** grammars, generate the *recursively enumerable* languages, automata equals *Turing machines*
- **context-sensitive** or **type-1** grammars, generate the *context-sensitive* languages, automata equals *Linear Bounded Automata*
- **context-free** or **type-2** grammars, generate the *context-free* languages, automata equals *Pushdown Automata*
- **regular** or **type-3** grammars, generate the *regular* languages, automata equals *Finite-State Automata*

The Chomsky Hierarchy

- A system of grammars $G = (N, T, P, S)$
- T is a set of symbols called terminal symbols.
  Also called the alphabet Σ
- N is a set of non-terminals, where $N \cap T = \emptyset$
  Some notation: $\alpha, \beta, \gamma \in (N \cup T)^*$
  N is sometimes called the set of variables V
- P is a set of production rules that provide a finite description of an infinite set of strings (a language)
- S is the start non-terminal symbol (similar to the start state in a FSA)

Languages

- Language defined by G: $L(G)$
  - $L(G)$: set of strings $w \in T^*$ derived from S
  - $S \Rightarrow^+ w$ (derives in 1 or more steps using rules in P)
  - w is a sentence of G
  - Sentential form: $S \Rightarrow^+ \alpha$ and α contains a mix of terminals and non-terminals
- Two grammars G_1 and G_2 are equivalent if $L(G_1) = L(G_2)$

The Chomsky Hierarchy: $G = (N, T, P, S)$ where, $\alpha, \beta, \gamma \in (N \cup T)^*$

- **unrestricted** or **type-0** grammars: $\alpha \rightarrow \gamma$, such that $\alpha \neq \epsilon$
- **context-sensitive** or **type-1** grammars: $\alpha \rightarrow \gamma$, where $|\gamma| \geq |\alpha|$
  CSG Normal Form: $\alpha A \beta \rightarrow \alpha \gamma \beta$, such that $\gamma \neq \epsilon$ and $S \rightarrow \epsilon$ if $\epsilon \in L(G)$
- **context-free** or **type-2** grammars: $A \rightarrow \gamma$
- **regular** or **type-3** grammars: $A \rightarrow a\ B$ or $A \rightarrow a$

Examples of Languages in the Chomsky Hierarchy

- **context-sensitive** grammars: 0^i, i is a prime number
- **indexed** grammars: $0^n 1^n 2^n \ldots m^n$, for any fixed m and $n \geq 0$
- **context-free** grammars: $0^n 1^n$ for $n \geq 0$;
  also $\{0^n 1^n 2^m\} \cup \{0^m 1^n 2^n\}$ which is *inherently* ambiguous, i.e. no unambiguous CFG exists!
- **deterministic context-free** grammars: $S' \rightarrow S\ c$, $S \rightarrow S\ A \mid A$, $A \rightarrow a\ S\ b \mid ab$: the language of "balanced parentheses"
- **regular** grammars: $(0|1)^* 00 (0|1)^*$

Language	*Automaton*	*Grammar*	*Recognition*	*Dependency*
Recursively Enumerable Languages	Turing Machine	Unrestricted Baa → A	Undecidable	Arbitrary
Context-Sensitive Languages	Linear-Bounded	Context-Sensitive At → aA	NP-Complete	Crossing
Context-Free Languages	Pushdown (stack)	Context-Free S → gSc	Polynomial	Nested
Regular Languages	Finite-State Machine	Regular A → cA	Linear	Strictly Local

Complexity of Parsing Algorithms

- Given grammar G and input x, provide algorithm for: Is $x \in L(G)$?
  - **unrestricted**: undecidable
  - **context-sensitive**: NSPACE(n) – linear non-deterministic space
  - **indexed** grammars: NP-Complete
  - **context-free**: $\mathcal{O}(n^3)$
  - **deterministic context-free**: $\mathcal{O}(n)$
  - **regular** grammars: $\mathcal{O}(n)$

Summary

- Aspects of PL structure cannot be represented by FSAs
- We can show that a language is not regular.
- If such a language is needed for our programming language then we have to use something more powerful than a regular language
- Chomsky hierarchy: from FSAs to Turing machines
- Context-free grammars (seems sufficient for PLs) but problems with ambiguity