Probabilistic CFG (PCFG)

\[
\begin{align*}
S & \rightarrow \ NP \ VP \quad 1 \\
VP & \rightarrow \ V \ NP \quad 0.9 \\
VP & \rightarrow \ VP \ PP \quad 0.1 \\
PP & \rightarrow \ P \ NP \quad 1 \\
NP & \rightarrow \ NP \ PP \quad 0.25 \\
NP & \rightarrow \ Calvin \quad 0.25 \\
NP & \rightarrow \ monsters \quad 0.25 \\
NP & \rightarrow \ school \quad 0.25 \\
V & \rightarrow \ imagined \quad 1 \\
P & \rightarrow \ in \quad 1
\end{align*}
\]

\[
P(\text{input}) = \sum_{\text{tree}} P(\text{tree} \mid \text{input})
\]

\[
P(\text{Calvin imagined monsters in school}) = ?
\]

Notice that \( P(\text{VP} \rightarrow \ V \ \text{NP}) + P(\text{VP} \rightarrow \ VP \ PP) = 1.0 \)
Probabilistic CFG (PCFG)

\[ P(\text{Calvin imagined monsters in school}) = ? \]

\[
(S \ (NP \ Calvin) \\
\quad (VP \ (V \ imagined) \\
\quad \quad (NP \ (NP \ monsters) \\
\quad \quad \quad (PP \ (P \ in) \\
\quad \quad \quad \quad (NP \ school)))))
\]

\[
(S \ (NP \ Calvin) \\
\quad (VP \ (VP \ (V \ imagined) \\
\quad \quad (NP \ monsters)) \\
\quad \quad (PP \ (P \ in) \\
\quad \quad \quad (NP \ school))))
\]
(S (NP Calvin)
   (VP (V imagined)
      (NP (NP monsters)
         (PP (P in)
            (NP school))))))

\[ P(tree_1) = P(S \rightarrow NP \ VP) \times P(NP \rightarrow Calvin) \times P(VP \rightarrow V \ NP) \times \]
\[ P(V \rightarrow imagined) \times P(NP \rightarrow NP PP) \times P(NP \rightarrow monsters) \times \]
\[ P(PP \rightarrow P \ NP) \times P(P \rightarrow in) \times P(NP \rightarrow school) \]
\[ = 1 \times 0.25 \times 0.9 \times 1 \times 0.25 \times 0.25 \times 1 \times 1 \times 0.25 = .003515625 \]
Probabilistic CFG (PCFG)

(S (NP Calvin)
 (VP (VP (V imagined)
   (NP monsters))
 (PP (P in)
   (NP school))))

\[ P(\text{tree}_2) = P(S \to NP \ VP) \times P(NP \to Calvin) \times P(VP \to VP PP) \times P(VP \to V NP) \times P(V \to imagined) \times P(NP \to monsters) \times P(PP \to P NP) \times P(P \to in) \times P(NP \to school) \]
\[ = 1 \times 0.25 \times 0.1 \times 0.9 \times 1 \times 0.25 \times 1 \times 1 \times 0.25 = 0.00140625 \]
**Probabilistic CFG (PCFG)**

\[ P(\text{Calvin imagined monsters in school}) = P(tree_1) + P(tree_2) \]
\[ = 0.003515625 + 0.00140625 \]
\[ = 0.004921875 \]

Most likely tree is \( tree_1 = \arg \max_{tree} P(tree | input) \)

(S (NP Calvin)
  (VP (V imagined)
    (NP (NP monsters)
      (PP (P in)
        (NP school))))
)

(S (NP Calvin)
  (VP (VP (V imagined)
      (NP monsters))
    (PP (P in)
      (NP school))))
PCFG

- Central condition: $\sum_\alpha P(A \rightarrow \alpha) = 1$
- Called a *proper* PCFG if this condition holds
- Note that this means $P(A \rightarrow \alpha) = P(\alpha \mid A) = \frac{f(A,\alpha)}{f(A)}$
- $P(T \mid S) = \frac{P(T,S)}{P(S)} = P(T, S) = \prod_i P(RHS_i \mid LHS_i)$
What is the PCFG that can be extracted from this single tree:

(S (NP (Det the) (NP man))
   (VP (VP (V played)
       (NP (Det a) (NP game)))
   (PP (P with)
       (NP (Det the) (NP dog))))

How many different rhs $\alpha$ exist for $A \rightarrow \alpha$ where $A$ can be $S$, $NP$, $VP$, $PP$, $Det$, $N$, $V$, $P$
We can do this with multiple trees. Simply count occurrences of CFG rules over all the trees.

A repository of such trees labelled by a human is called a TreeBank.
Ambiguity

- Part of Speech ambiguity
  - saw → noun
  - saw → verb

- Structural ambiguity: Prepositional Phrases
  - I saw (the man) with the telescope
  - I saw (the man with the telescope)

- Structural ambiguity: Coordination
  - a program to promote safety in ((trucks) and (minivans))
  - a program to promote ((safety in trucks) and (minivans))
  - ((a program to promote safety in trucks) and (minivans))
Ambiguity ← attachment choice in alternative parses
Parsing as a machine learning problem

- \( S = \) a sentence
- \( T = \) a parse tree

A statistical parsing model defines \( P(T \mid S) \)

- Find best parse: \( \arg \max_T P(T \mid S) \)

- \( P(T \mid S) = \frac{P(T, S)}{P(S)} = P(T, S) \)

- Best parse: \( \arg \max_T P(T, S) \)

- e.g. for PCFGs: \( P(T, S) = \prod_{i=1}^{n} P(RHS_i \mid LHS_i) \)
Prepositional Phrases

- noun attach: *I bought the shirt with pockets*
- verb attach: *I washed the shirt with soap*
- As in the case of other attachment decisions in parsing: it depends on the meaning of the entire sentence – needs world knowledge, etc.
- Maybe there is a simpler solution: we can attempt to solve it using heuristics or associations between words
Structure Based Ambiguity Resolution

- Right association: a constituent (NP or PP) tends to attach to another constituent immediately to its right (Kimball 1973)
- Minimal attachment: a constituent tends to attach to an existing non-terminal using the fewest additional syntactic nodes (Frazier 1978)
- These two principles make opposite predictions for prepositional phrase attachment
- Consider the grammar:

\[
\begin{align*}
VP & \rightarrow V \ NP \ PP \\
NP & \rightarrow NP \ PP
\end{align*}
\]

For input: \( I [VP \ saw [NP \ the \ man \ldots [PP \ with \ the \ telescope ]], \)
RA predicts that the PP attaches to the NP, i.e. use rule (2), and MA predicts V attachment, i.e. use rule (1)
Structure Based Ambiguity Resolution

- Garden-paths look structural:
  *The emergency crews hate most is domestic violence*

- Neither MA or RA account for more than 55% of the cases in real text

- Psycholinguistic experiments using eyetrackning show that humans resolve ambiguities as soon as possible in the left to right sequence using the words to disambiguate

- Garden-paths are caused by a combination of lexical and structural effects:
  *The flowers delivered for the patient arrived*
Learning Prepositional Phrase Attachment: Annotated Data

<table>
<thead>
<tr>
<th>v</th>
<th>n1</th>
<th>p</th>
<th>n2</th>
<th>Attachment</th>
</tr>
</thead>
<tbody>
<tr>
<td>join</td>
<td>board</td>
<td>as</td>
<td>director</td>
<td>V</td>
</tr>
<tr>
<td>is</td>
<td>chairman</td>
<td>of</td>
<td>N.V.</td>
<td>N</td>
</tr>
<tr>
<td>using</td>
<td>crocidolite</td>
<td>in</td>
<td>filters</td>
<td>V</td>
</tr>
<tr>
<td>bring</td>
<td>attention</td>
<td>to</td>
<td>problem</td>
<td>V</td>
</tr>
<tr>
<td>is</td>
<td>asbestos</td>
<td>in</td>
<td>products</td>
<td>N</td>
</tr>
<tr>
<td>making</td>
<td>paper</td>
<td>for</td>
<td>filters</td>
<td>N</td>
</tr>
<tr>
<td>including</td>
<td>three</td>
<td>with</td>
<td>cancer</td>
<td>N</td>
</tr>
<tr>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
<td>:</td>
</tr>
</tbody>
</table>
### Prepositional Phrase Attachment

<table>
<thead>
<tr>
<th>Method</th>
<th>Accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>Always noun attachment</td>
<td>59.0</td>
</tr>
<tr>
<td>Most likely for each preposition</td>
<td>72.2</td>
</tr>
<tr>
<td>Average Human (4 head words only)</td>
<td>88.2</td>
</tr>
<tr>
<td>Average Human (whole sentence)</td>
<td>93.2</td>
</tr>
</tbody>
</table>
Let 1 represent noun attachment.
We want to compute probability of noun attachment: 
\[ p(1 \mid v, n1, p, n2). \]
Probability of verb attachment is 1 – \( p(1 \mid v, n1, p, n2). \)
Back-off Smoothing

1. If \( f(v, n1, p, n2) > 0 \) and \( \hat{\rho} \neq 0.5 \)
   \[
   \hat{\rho}(1 \mid v, n1, p, n2) = \frac{f(1, v, n1, p, n2)}{f(v, n1, p, n2)}
   \]

2. Else if \( f(v, n1, p) + f(v, p, n2) + f(n1, p, n2) > 0 \) and \( \hat{\rho} \neq 0.5 \)
   \[
   \hat{\rho}(1 \mid v, n1, p, n2) = \frac{f(1, v, n1, p) + f(1, v, p, n2) + f(1, n1, p, n2)}{f(v, n1, p) + f(v, p, n2) + f(n1, p, n2)}
   \]

3. Else if \( f(v, p) + f(n1, p) + f(p, n2) > 0 \)
   \[
   \hat{\rho}(1 \mid v, n1, p, n2) = \frac{f(1, v, p) + f(1, n1, p) + f(1, p, n2)}{f(v, p) + f(n1, p) + f(p, n2)}
   \]

4. Else if \( f(p) > 0 \)
   \[
   \hat{\rho}(1 \mid v, n1, p, n2) = \frac{f(1, p)}{f(p)}
   \]

5. Else \( \hat{\rho}(1 \mid v, n1, p, n2) = 1.0 \)
Results: 84.5% accuracy with the use of some limited word classes for dates, numbers, etc.

Using complex word classes taken from WordNet (which we shall be looking at later in this course) increases accuracy to 88% (Stetina and Nagao 1998)

We can improve on parsing performance with Probabilistic CFGs by using the insights taken from PP attachment.

Modify the PCFG model to be sensitive to words and other context-sensitive features of the input.

And generalizing to other kinds of attachment problems, like coordination or deciding which constituent is an argument of a verb.
Some other studies

- **Toutanova, Manning, and Ng, 2004:**
  use sophisticated smoothing model for PP attachment
  86.18% with words & stems; with word classes: 87.54%

- **Merlo, Crocker and Berthouzoz, 1997:**
  test on multiple PPs, generalize disambiguation of 1 PP to 2-3 PPs
  14 structures possible for 3PPs assuming a single verb: all 14 are attested in the Treebank
  same model as CB95; but generalized to dealing with upto 3PPs
  1PP: 84.3%  2PP: 69.6%  3PP: 43.6%

  Note that this is still not the real problem faced in parsing natural language
Adding Lexical Information to PCFG
Adding Lexical Information to PCFG (Collins 99, Charniak 00)

\[
P_h(VB \mid VP, \text{indicated}) \times P_I(\text{STOP} \mid VP, VB, \text{indicated}) \times \\
P_r(\text{NP} \mid \text{difference}) \mid VP, VB, \text{indicated}) \times \\
P_r(\text{PP} \mid \text{in}) \mid VP, VB, \text{indicated}) \times \\
P_r(\text{STOP} \mid VP, VB, \text{indicated})
\]
Evaluation of Parsing

Consider a candidate parse to be evaluated against the truth (or gold-standard parse):

candidate: (S (A (P this) (Q is)) (A (R a) (T test)))
gold: (S (A (P this)) (B (Q is) (A (R a) (T test))))

In order to evaluate this, we list all the constituents:

<table>
<thead>
<tr>
<th>Candidate</th>
<th>Gold</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,4,S)</td>
<td>(0,4,S)</td>
</tr>
<tr>
<td>(0,2,A)</td>
<td>(0,1,A)</td>
</tr>
<tr>
<td>(2,4,A)</td>
<td>(1,4,B)</td>
</tr>
<tr>
<td></td>
<td>(2,4,A)</td>
</tr>
</tbody>
</table>

Skip spans of length 1 which would be equivalent to part of speech tagging accuracy.

Precision is defined as $\frac{\#\text{correct}}{\#\text{proposed}} = \frac{2}{3}$ and recall as $\frac{\#\text{correct}}{\#\text{in gold}} = \frac{2}{4}$.

Another measure: crossing brackets,

candidate: [ an [incredibly expensive] coat ] (1 CB)
gold: [ an [incredibly [expensive coat]]]
## Evaluation of Parsing

<table>
<thead>
<tr>
<th>Metric</th>
<th>Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bracketing recall $R$</td>
<td>$\frac{\text{num of correct constituents}}{\text{num of constituents in the goldfile}}$</td>
</tr>
<tr>
<td>Bracketing precision $P$</td>
<td>$\frac{\text{num of correct constituents}}{\text{num of constituents in the parsed file}}$</td>
</tr>
<tr>
<td>Complete match</td>
<td>% of sents where recall &amp; precision are both 100%</td>
</tr>
<tr>
<td>Average crossing</td>
<td>$\frac{\text{num of constituents crossing a goldfile constituent}}{\text{num of sents}}$</td>
</tr>
<tr>
<td>No crossing</td>
<td>% of sents which have 0 crossing brackets</td>
</tr>
<tr>
<td>2 or less crossing</td>
<td>% of sents which have $\leq 2$ crossing brackets</td>
</tr>
</tbody>
</table>
## Statistical Parsing Results

<table>
<thead>
<tr>
<th>System</th>
<th>( \leq 40wds )</th>
<th>( \leq 40wds )</th>
<th>( \leq 100wds )</th>
<th>( \leq 100wds )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>P</td>
<td>R</td>
<td>P</td>
<td>R</td>
</tr>
<tr>
<td>(Magerman 95)</td>
<td>84.9</td>
<td>84.6</td>
<td>84.3</td>
<td>84.0</td>
</tr>
<tr>
<td>(Collins 99)</td>
<td>88.5</td>
<td>88.7</td>
<td>88.1</td>
<td>88.3</td>
</tr>
<tr>
<td>(Charniak 97)</td>
<td>87.5</td>
<td>87.4</td>
<td>86.7</td>
<td>86.6</td>
</tr>
<tr>
<td>(Ratnaparkhi 97)</td>
<td></td>
<td></td>
<td>86.3</td>
<td>87.5</td>
</tr>
<tr>
<td>(Charniak 99)</td>
<td>90.1</td>
<td>90.1</td>
<td>89.6</td>
<td>89.5</td>
</tr>
<tr>
<td>(Collins 00)</td>
<td>90.1</td>
<td>90.4</td>
<td>89.6</td>
<td>89.9</td>
</tr>
<tr>
<td>Voting (HB99)</td>
<td>92.09</td>
<td>89.18</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Practical Issues: Beam Thresholding and Priors

- Probability of nonterminal $X$ spanning $j \ldots k$: $N[X, j, k]$
- Beam Thresholding compares $N[X, j, k]$ with every other $Y$ where $N[Y, j, k]$
- But what should be compared?
- Just the *inside probability*: $P(X \Rightarrow t_j \ldots t_k)$?
  written as $\beta(X, j, k)$
- Perhaps $\beta(\text{FRAG}, 0, 3) > \beta(\text{NP}, 0, 3)$, but NPs are much more likely than FRAGs in general
The correct estimate is the *outside probability*:

\[ P(S \Rightarrow t_1 \ldots t_{j-1} X t_{k+1} \ldots t_n) \]

written as \( \alpha(X, j, k) \)

Unfortunately, you can only compute \( \alpha(X, j, k) \) efficiently after you finish parsing and reach \((S, 0, n)\)
Practical Issues: Beam Thresholding and Priors

- To make things easier we multiply the prior probability $P(X)$ with the inside probability.
- In beam Thresholding we compare every new insertion of $X$ for span $j, k$ as follows:
  Compare $P(X) \cdot \beta(X, j, k)$ with the most probable $Y$
  $P(Y) \cdot \beta(Y, j, k)$
- Assume $Y$ is the most probable entry in $j, k$, then we compare
  $\text{beam} \cdot P(Y) \cdot \beta(Y, j, k)$
  $P(X) \cdot \beta(X, j, k)$

- If (4) < (3) then we prune $X$ for this span $j, k$
- beam is set to a small value, say 0.001 or even 0.01.
- As the beam value increases, the parser speed increases (since more entries are pruned).
- A simpler (but not as effective) alternative to using the beam is to keep only the top $K$ entries for each span $j, k$
Experiments with Beam Thresholding

![Graph showing parsing time vs sentence length for different beam thresholds.](image-url)