Answer questions worth 20 points

(1) (5 points) Which of the following questions has the solution $\frac{15!}{10!}$? Justify.
(a) How many ways can a committee of 5 people be formed with 15 people? $\binom{15}{5}$
(b) How many bit strings of length 10 begin with the sequence 01101? $2^5$
(c) How many ways are there to distribute 10 identical objects into 6 distinct boxes? $\binom{10+6-1}{6-1}$
(d) How many ways are there to distribute 15 different books to 5 children? $\binom{15,5}$ without order, or $P(15,5)$ with order
(e) How many ways can 10 out of 15 people be arranged in a lineup? $P(15,10)$

The solution to Question 1 is either none or (d)

(2) (5 points) We are given $n$ distinct objects. We want to arrange objects in a circle where arrangements are considered the same if one can be obtained from the other by rotation. Determine how many ways
(a) all these objects can be arranged in a circle. $P(n-1,n-1)$
(b) $r$ of these objects can be arranged in a circle. $\binom{n,r}.P(r-1,r-1)$

(3) (5 points) In a given race, there are twenty countries (including Canada and USA) competing for eight different medals. How many different ways are there to award these medals if
(a) there is no restriction? $P(20,8)$ w/o repetition; with repetition doesn’t make any sense.
(b) Canada and USA must get one of the top four spots? $P(4,2)P(18,6)$ w/o repetition;

(4) (5 points) An apartment building has three floors and four families can live on each floor. There are twelve families who will occupy the building. In how many ways one can assign 12 families if
(a) there are no restrictions? $P(12,12) = 12!$
(b) two particular families (say, families X and Y) must be assigned to one floor? $\binom{3,1}$ ways to assign X and Y to a floor; once a floor is assigned, there are $P(4,2)$ ways to assign 2 rooms on the floor; remaining spots
can be assigned 10! ways; total ways is P(3,1)*P(4,2)*10! 

(c) family X and family Y must be assigned to different floors? This number is 12! - P(3,1)*P(4,2)*10!. Give full marks to this question if they figured out the way even though question (b) is wrongly computed.

(5) (5 points) Find the number of integral solutions to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 21$ subject to the conditions

(a) $x_i \geq 0, i = 1, 2, 3, 4, 5, 6$. $C(21+6-1,6-1)$

(b) $x_3 \geq 5, x_i \geq 1, i = 1, 2, 4, 5, 6$. $C(11+6-1)$

(c) $0 \leq x_1 \leq 3, x_i \geq 1, i = 2, 3, 4, 5, 6$. Ans = number of integral solutions to $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 21, x_1 \geq 0, x_i \geq 1, i = 2, 3, 4, 5, 6$ minus the number of integral solutions to $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 21, x_1 \geq 4, x_i \geq 1, i = 2, 3, 4, 5, 6 = C(16 + 6 - 1, 6 - 1) - C(12 + 6 - 1, 6 - 1)$. 