The Dichromatic Object Colour Solid

Alexander LOGVINENKO[1], Pouya BASTANI[2], and Brian FUNT[2]
[1] Department of Life Sciences, Glasgow Caledonian University
[2] School of Computing Science, Simon Fraser University

colour imaging

ABSTRACT

The set of all possible cone excitation triplets from reflecting surfaces under a given illuminant forms a volume in cone excitation space known as the *object-colour solid* (OCS). An important task in Color Science is to specify the precise geometry of the OCS as defined by its boundary. Schrödinger claimed that the *optimal* reflectances that map to the boundary of the OCS take on values of 0 or 1 only, with no more than two wavelength transitions. Although this popularly accepted assertion is, by and large, correct and holds under some restricted conditions (e.g., it holds for the CIE colour matching functions), as far as the number of transitions is concerned, it has been shown not to hold in general. As a result, the Schrödinger optimal reflectances provide only an approximation to the true OCS. For the case of dichromatic vision, we compare the true and approximate OCS by computing the set of true optimal reflectances, and find that they differ significantly.

1. INTRODUCTION

Light reaching the eye from a reflecting surface generates a triplet of cone responses. Formally, the *color signal* triplet $(\phi_1(x),\phi_2(x),\phi_3(x))$ can be written as the integral over the visible spectrum:

$$\phi_i(x) = \int_{\lambda_{\min}}^{\lambda_{\max}} x(\lambda) I(\lambda) s_i(\lambda)\, d\lambda,$$

where $x(\lambda)$ is the spectral reflectance of the surface, $I(\lambda)$ is the spectral power distribution of the illuminant, and $s_i(\lambda)$ is the spectral sensitivity of the i^{th} sensor. The set of all such color signals, represented as points in 3D space with Cartesian coordinates (ϕ_1,ϕ_2,ϕ_3), forms a volume referred to as the *object-colour solid* (Wyszecki and Stiles 2000). This volume can be precisely described by its boundary, formed by the so-called *optimal reflectances*, reflectance functions whose corresponding colour signals map to the boundary of the OCS. Thus, determining the set of reflectances that are *optimal* under a given illuminant is a crucial step in defining the geometry of the OCS.

The first part of Schrödinger's assumption (1920) about optimal reflectances is correct; namely, that they are *elementary step functions* taking on values of 0 or 1 only. However, the restriction that they contain only 2 or fewer transitions holds only under restricted conditions. Nonetheless, it has been a commonly held view (Koenderink 2010, MacAdam 1935, Wyszecki and Stiles 2000) that optimal reflectances in general have no more than 2 wavelength transitions. This 2-transition assumption has been shown to be incorrect (Logvinenko and Levin 2013, Maximov 1984, West and Brill 1983) and any colour solid based on it is only an approximation.

As shown by Logvinenko and Levin (2013) for continuous linearly independent spectral sensitivity functions, $s_1(\lambda),\ldots,s_n(\lambda)$ the transition wavelengths $\lambda_1,\ldots,\lambda_m$ of an optimal reflectance function are determined by the zero-crossings of the following equation, where $k_1,\ldots,k_n$

are arbitrary real numbers, at least one of which is not zero:

$$g(\lambda) = k_1 s_1(\lambda) + \cdots + k_n s_n(\lambda) = 0 \qquad (1)$$

While the analogue of Schrodinger's assumption in the case of dichromatic vision would state that the optimal reflectances have at most 1 transition (the *1-transition assumption)*, our results reveal that for tritanopic vision the number of transitions can be up to 3. By computing transition wavelengths of optimal reflectance functions using equation (1) the precise loci of optimal stimuli, and hence the boundary of the OCS, can be determined. In this paper, we compare the difference between the true dichromatic OCS and its 1-transition approximation and find it to be significant.

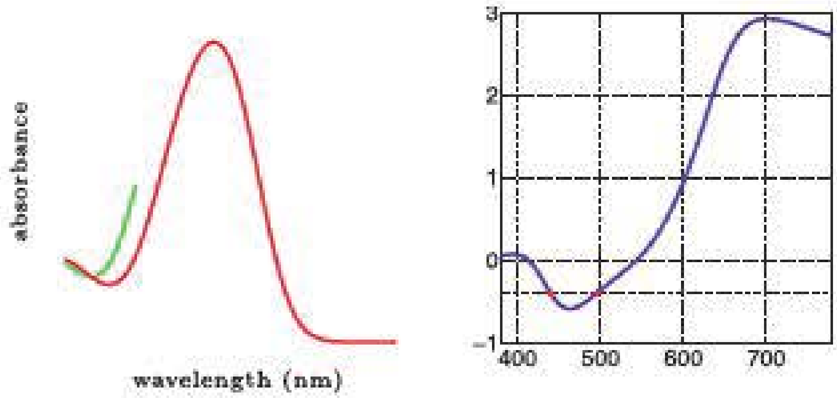

Figure 1: Left: normalized spectral sensitivity of M- and L-cone photopigments with peak sensitivities at 530 and 560 nm, respectively. Right: log(L/M) cone response ratio. The dashed line is one example of a horizontal line that intersects the curve more than once.

2. DICHROMATIC OBJECT-COLOUR SOLID

In this section, we consider the case of tritanopic vision, where S-cones are not present. To study the 2-dimensional OCS for this dichromatic visual system we first note that the cone sensitivity functions $s_i(\lambda)$ can be factored as $s_i(\lambda) = t(\lambda)p_i(\lambda)$, where $t(\lambda)$ is the transmittance spectrum of the ocular media and is the spectral absorption of the ith photopigment. Since $t(\lambda) > 0$ over the visible spectrum, the zero crossings of Equation (1) are the same as those of the following equation:

$$k_1 p(\lambda;\, \lambda_1^{\max}) + \cdots + k_n p(\lambda;\, \lambda_n^{\max}) = 0, \qquad (2)$$

where $p(\lambda;\, \lambda_i^{\max})$ is the i^{th} cone-photopigment absorbance spectrum with peak absorbance at $\lambda_i^{\max}$. The following results are based on Govardovskii et al. (2000) model of absorbance spectra for cone photopigments. In accordance with electrophysiological studies of the cones in the human retina (Schnapf and Schneewels 1999), we set the model parameter for photopigment optical density to 0.3, and use peak absorbance wavelengths of 530 nm and 560 nm for the M- and L-cones (cones with peak sensitivity in the middle, and long wavelength range of the visible spectrum), respectively. Figure 1 (left) shows the spectral sensitivities of the M- and L-cone photopigments obtained using the Govardovskii model with the above parameter values. Throughout this paper we will use $\lambda_{min} = 380$ nm and $\lambda_{max} =$ 780 nm.

For such a dichromatic visual system, Equation (2) with $n = 2$ can be re-written as $p(\lambda;\, \lambda_L^{\max})/p(\lambda;\, \lambda_M^{\max}) = k,$ where is an arbitrary real number. Figure 1 (right) shows a semi-logarithmic plot of the ratio of L-cone to M-cone responses. As is clear from the picture, there are values of for which a horizontal line intersects the curve at more than 1 location, indicating that the above equation has more than one zero-crossing in the visible spectrum for certain values of , thus violating the 1-transition assumption. The dashed line, for instance, intersects the curve at $\lambda_1 = 439$ nm and at $\lambda_2 = 496$ nm, resulting in a pair of complementary optimal reflectance functions $x(\lambda)$ and $x'(\lambda) = \lambda - x(\lambda)$. Reflectance $x(\lambda)$ is zero everywhere except for $x \in [439, 496]$, where it is one. In fact, each k in Equation (1) determines zero-crossings, which in turn determine a pair of complementary optimal reflectance functions that map to points on the boundary of the object-colour solid that are symmetrically situated about its center. Figure 2 (left) shows the boundary of the true OCS and its 1-transition approximation **obtained under the flat-spectrum illuminant,** $I(\lambda)$. **Note that the latter, shown dashed, lacks the convex geometry that the OCS is known to possess** (Logvinenko and Levin, 2013).

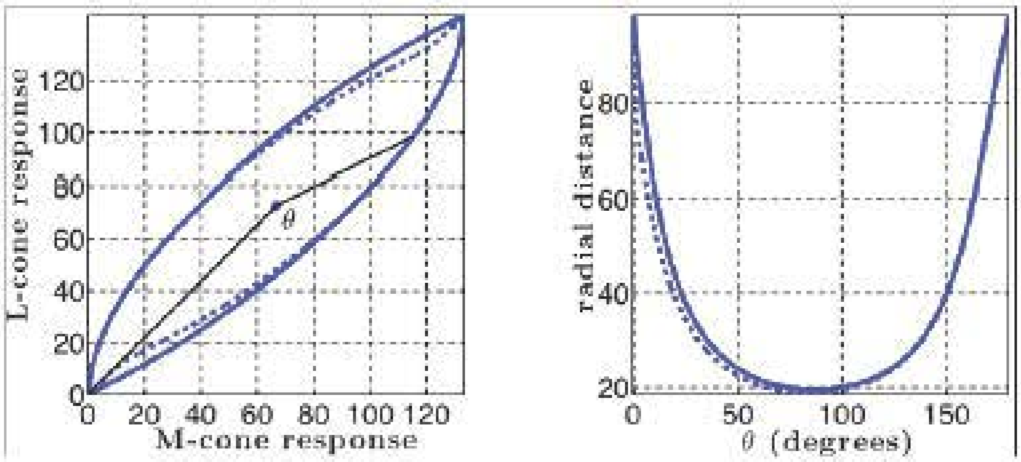

Figure 2: Left: boundary of the true tritanopic OCS (solid curve) and its 1-transition approximation (dashed curve). Right: radial distance from the center to the boundary of the dichromatic OCS (solid curve) versus its 1-transition approximation (dashed curve).

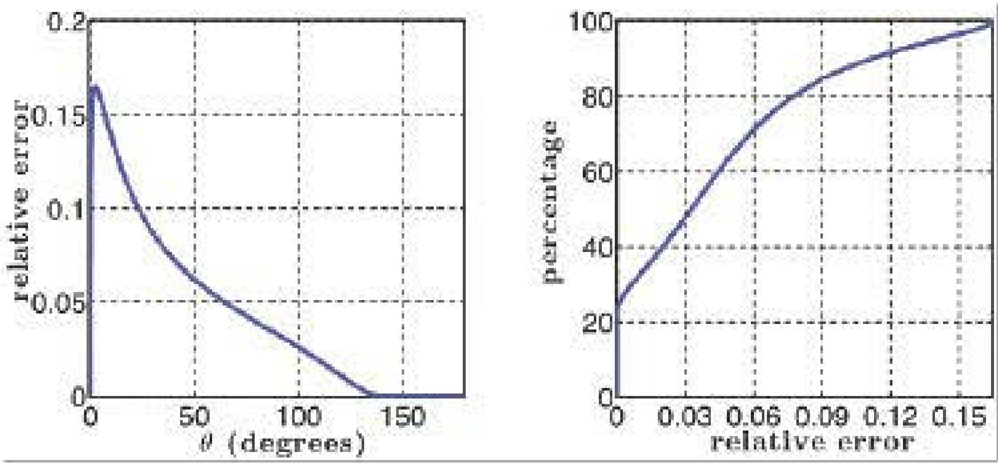

Figure 3: Left: relative error, as a function of polar angle, in the 1-transition approximation of the tritanopic OCS as measured by the ratio of radial distances to the boundary. Right: cumulative distribution plot of the relative error.

This boundary can be described by the radial distance from the center as a function of the polar angle measured counter-clockwise with for the direction to the origin (see Figure 2 left). Figure 2 (right) shows the plot of the radial distance to the boundary of the tritanope's OCS and its 1-transition approximation (since the OCS is symmetric about its center, only is considered). The plot of the relative error in the radial distance of the approximate and true OCSs (Figure 3, left) indicates that the difference can be as high as 16%, though it decreases rapidly with Figure 3 (right), showing a cumulative distribution plot of the relative difference, gives a good idea of the significance of the error in computing optimal stimuli based on the 1-transition approximation. As can be seen, only 23% of optimal reflectance functions are correctly predicted (i.e., have a relative error of zero) using the 1-transition assumption, and 13% of the radial distances are underestimated by 10% or more.

CONCLUSION

The true dichromatic OCS was computed and compared to its approximation based on the assumption that the optimal reflectances are 1-transition functions taking values of 0 and 1 only. It was found that the optimal reflectances for the dichromatic OCS include not only 1-transition reflectances, but 2- and 3-transition reflectances as well. A quantitative comparison between the true and approximated tritanopic OCS was performed using the radial distances to the boundary of the OCS. The approximation error due to the 1-transition assumption was shown to be as large as 16%, with over 77% of optimal stimuli computed inaccurately.

REFERENCES

Govardovskii, V. I., N. Fyhrquist, T. Reuter, D.G. Kuzmin and K. Donner, 2000. In search of the visual pigment template. *Visual Neuroscience*: 17, 509-528.

Koenderink, J.J., 2010. *Color for the sciences*. Cambridge, Massachusetts; London, England: The MIT Press.

Logvinenko, A.D., 2009. An object-colour space. *Journal of Vision*, 9 (11), 1-23.

Logvinenko, A.D., and V.L. Levin, 2013. *Foundations of Colour Science* (in preparation).

MacAdam, D.L., 1935. The theory of the maximum visual efficiency of colored materials. *Journal of the Optical Society of America*: 25, 249-252.

Maximov, V.V, 1984. Transformatsii tsveta pri izmenenii osvescheniya [Transformations of colour under illumination changes]. Moscow: Nauka.

Schnapf, J. L., and D.M. Schneewels, 1999. Electrophysiology of cone photoreceptors in the primate retina. In K.R. Gegenfurtner and L.T. Sharpe (Eds.), *Color vision: From genes to perception* (pp. 103-112). Cambridge: Cambridge University Press.

Schrodinger, E., 1920. Theorie der Pigmente von grosster Leuchtkraft. *Annalen der Physik*, 62, 603-622.

West, G., and M.H. Brill, 1983. Conditions under which Schrodinger object colors are optimal. *Journal of the Optical Society of America*, 73, 1223-1225.

Wyszecki, G., and W.S. Stiles, 2000. *Color science: Concepts and methods, quantitative data and formulae* (2nd ed.) New York: John Wiley and Sons.

Address: Prof. Alexander D. Logvinenko, Department of Life Sciences
Glasgow Caledonian University, City Campus, Cowcaddens Rd, Glasgow, G4 0BA, UK
E-mails: a.logvinenko@gcu.ac.uk