Quaternion Color Curvature

Lilong Shi, Brian Funt and Ghassan Hamarneh; Simon Fraser University; Vancouver, Canada

Abstract

In this paper we propose a novel approach to measuring curvature in color or vector-valued images (up to 4-dimensions) based on quaternion singular value decomposition of a Hessian matrix. This approach generalizes the existing scalar-image curvature approach which makes use of the eigenvalues of the Hessian matrix [1]. In the case of vector-valued images, the Hessian is no longer a 2D matrix but rather a rank 3 tensor. We use quaternion curvature to derive vesselness measure for tubular structures in color or vector-valued images by extending Frangi's [1] vesselness measure for scalar images. Experimental results show the effectiveness of quaternion color curvature in generating a vesselness map.

Introduction

Hessian-based methods have been widely used from curvature measures to feature detection [1-10]. The Hessian matrix describes the second-order structure of gray-level variations around each pixel of the image. There are two main categories where a Hessian matrix is used. First, the Hessian and the related second-moment matrix have been applied in several operators (e.g., the Harris [11], Harris-affine [12], and Hessian-affine [10] detectors) to find "interest" points where the local image geometry changes in more than one direction. Hessian-based blob detector in color space is proposed in [5]. Second, since the eigenvalues of the Hessian matrix at a pixel measure the principal curvatures of the image intensity surface, it can be used to detect tubular (linear, vessel-like) structures, which is useful in many applications [1,3,6-9]. By smoothing with Gaussian kernels of various sizes, the normalized second-order derivatives indicate the scale and orientation of vessels. Vesselness is measured by a large curvature in the cross-sectional direction and a small curvature along the vessel. By eigen-analysis of the Hessian matrix, elongated objects (i.e., vessels) are detected wherever the first eigenvalue is positive (or negative) and prominent. This process generates a single response for both lines and edges, producing a clearer sketch of an image's structure than is usually provided by the magnitude of gradient.

Existing first-derivative point/blob detectors are applied to gray scale images. In the case of color images, the basic approach has been to compute the derivatives of each color channel separately, and then add them to produce the final result [5]. However, the first derivatives of a color edge can be in opposing directions, so the summation can lead to cancellation of the derivatives. The same situation occurs in second-derivative-based Hessian detectors. Existing Hessian-based curvature methods are also based on gray scale images, whether the luminance image, or a single color channel. For example, Hessian-based multi-scale segmentation or enhancement of vessels in retinal images has been extensively studied [1,3,6-7], where only the green channel is used.

To make use of the extra information in a color image, we use the quaternion representation of color to extend Hessian curvature measures to the color domain. In particular, we extend Frangi's [1] vesselness approach by estimating principle curvatures in RGB color space using quaternion operations. Sanqwine [13] introduced the quaternion representation of color. Since quaternions, which are an extension of the complex numbers, consist of one real component and three imaginary components, a color can be represented by a pure quaternion having a real component of zero, and imaginary components R, G and B. With colors encoded in quaternions, the entries of the Hessian matrix become quaternions that combine secondary derivatives from all color channels in their imaginary components. Quaternion singular value decomposition (QSVD) [13,14] can then be applied to the Hessian matrix in order to find the principle curvatures as described by the two non-negative, real-valued singular values. These singular values can be used to measure vesselness or other features.

The remainder of the paper is organized as follows. Section 2 reviews the necessary definitions of the Hessian matrix and its eigen-system for scalar images. Section 3 describes the quaternion-based approach to color curvature, and extends Frangi's vesselness measure to vector-valued images. Section 4 shows the experimental results, and Section 5 concludes the paper.

Curvature and Vesselness Measure

Viewing an image as an intensity surface, the local shape characteristics of the surface at a particular point can be described by the Hessian matrix. Lines (i.e., straight or nearly straight curvilinear features) and edges have high curvature in one direction and low curvature in the orthogonal direction, and this characteristic can be measured via the Hessian, *H*. For a 2D scalar image, *H* is a 2x2 matrix of the second derivatives of image *I*

$$H(\sigma) = \begin{bmatrix} \frac{\partial^2 I}{\partial x^2} & \frac{\partial^2 I}{\partial x \partial y} \\ \frac{\partial^2 I}{\partial y \partial x} & \frac{\partial^2 I}{\partial y^2} \end{bmatrix}. \qquad 1)$$

The four entries of *H* are the second-order partial derivatives of the scalar image *I* evaluated at the pixel $p = <x, y>$, and σ is the Gaussian scale of the partial derivatives.

The eigenvalues of *H* are called principal curvatures and are invariant under rotation. The eigenvectors of *H* can be used to define a coordinate system that is aligned with the dominant directions of curvature. Given the ordered eigenvalues of *H* such that $|\lambda_1|<|\lambda_2|$ with corresponding eigenvectors ($\boldsymbol{e}_1$, $\boldsymbol{e}_2$), the eigenvectors define an orthogonal coordinate system aligned with the direction of minimal $\boldsymbol{e}_1$ and maximal $\boldsymbol{e}_2$ curvature.

In the case of a vessel-like structure, $\boldsymbol{e}_1$ indicates the orientation of the vessel. Thus $\boldsymbol{e}_1$ represents the parallel curvature, and $\boldsymbol{e}_2$ the orthogonal curvature. As a vesselness measure for 2D images, Frangi [1] uses *H* to describe the curvature at each point in the image. The idea behind eigenvalue analysis of the Hessian is to extract the principal directions in which the local second-order structure of the image can be decomposed. Since this directly gives the direction of least curvature (along the vessel), application of several filters in multiple orientations is avoided.

Both eigenvalues play an important role in the vesselness measure. In particular, for a vessel we expect $|\lambda_1|<|\lambda_2|$, with $\lambda_2 < 0$ for bright vessels against a dark background, and $\lambda_2 > 0$ for the reverse. Finally the overall magnitude of the eigenvalues should be larger at vessels than in background regions. The Frangi filter combines these observations in the following two quantities

$$R_B = \frac{|\lambda_1|}{|\lambda_2|}, \tag{2}$$

$$S = \| H_\sigma \| = \sqrt{\lambda_1^2 + \lambda_2^2}, \tag{3}$$

Here, R_B is the *blobness* measure in 2D. It is maximized for highly blob-like structures and decreases as the difference between the parallel and orthogonal curvature increases. S is the norm of the Hessian matrix and measures the relative brightness/darkness of the structure. It should become large for vessels. In other words, it presents the "unlikelihood" that a pixel is from the background. These quantities are combined using exponentiation yielding a ''vesselness'' measure (for the bright-vessels-on-dark case) defined as follows:

$$V(\sigma) = \begin{cases} 0 & if\ \lambda_2 > 0, \\ e^{\frac{-R_B^2}{2\beta^2}}(1 - e^{\frac{-S^2}{2c^2}}) & otherwise \end{cases} \tag{4}$$

The constants β and c are parameters, which control the sensitivity of the filter to *blobness* and *backgroundness*.

Eigenvalues of the Color Hessian Matrix

As mentioned above, when the gradient of a color image is computed by adding up the first derivatives of the separate channels, the channel derivatives may point in opposing directions and cancel one another. DiZenzo [15] argues that a simple summation of the derivatives ignores the correlation between the channels. A similar problem arises in converting a color image to a luminance image in order to calculate its gradient. As a solution in the first-derivative case, DiZenzo[15] and Kass[16] proposed the color tensor by color gradient, but it does not generalize to the color Hessian matrix. The alternative of solving for the eigenvalues of the Hessian matrix separately in each color channel generates three pairs of eigenvalues, but these then do not immediately fit into the schema of Frangi's vesselness measure. Ming [5] used a weighted combination of Hessian matrices over HSI color channels to calculate a color Hessian. However, this approach does not eliminate the cancellation problem either. Our proposal, which uses the eigenvalues and eigenvectors of a color Hessian matrix based on quaternion singular value decomposition [13,14], overcomes the cancellation problem.

In the quaternion representation of a 2D color image, each pixel $p=<x,y>$ is represented by a quaternion number $Q=I_1 \cdot i + I_2 \cdot j + I_3 \cdot k$, where I_n (with n = 1,2,3) is the n^{th} channel of the input image, and i, j, and k are three imaginary bases. The quaternion representation of Hessian matrix H_Q is constructed as

$$H_Q(\sigma) = \begin{bmatrix} \frac{\partial^2 I_1}{\partial x^2} & \frac{\partial^2 I_1}{\partial x \partial y} \\ \frac{\partial^2 I_1}{\partial y \partial x} & \frac{\partial^2 I_1}{\partial y^2} \end{bmatrix} \cdot i + \begin{bmatrix} \frac{\partial^2 I_2}{\partial x^2} & \frac{\partial^2 I_2}{\partial x \partial y} \\ \frac{\partial^2 I_2}{\partial y \partial x} & \frac{\partial^2 I_2}{\partial y^2} \end{bmatrix} \cdot j + \begin{bmatrix} \frac{\partial^2 I_3}{\partial x^2} & \frac{\partial^2 I_3}{\partial x \partial y} \\ \frac{\partial^2 I_3}{\partial y \partial x} & \frac{\partial^2 I_3}{\partial y^2} \end{bmatrix} \cdot k \tag{5}$$

Quaternion Singular Value Decomposition (QSVD) is a generalization of SVD of real or complex numbers to quaternion numbers, inheriting similar properties [13,14]. By QSVD, a quaternion matrix can be decomposed into two unitary quaternion matrices, and one diagonal matrix consisting of real numbers. Therefore, existing SVD-based image processing algorithms for gray-scale images can be easily extended to color images using QSVD. Applications based on QSVD for color image compression and segmentation have been demonstrated [13,17]. Here, QSVD is applied to decompose the quaternion-valued matrix H_Q in Eqn. (5),

$$H_Q = V_Q^T \cdot \Lambda \cdot U_Q, \tag{6}$$

where V_Q and U_Q are two quaternion matrices of eigenvectors, and Λ is a real-valued diagonal matrix containing two non-negative singular values ξ_1 and ξ_2. Given the assumption that quaternion eigenvector corresponding to the smaller singular value of the Hessian points along the direction of minimal curvature, and that the larger singular value points along the direction of the maximum curvature, we can continue using Eqn. (4), but now as a color vesselness measure. It should be noted that the two singular values ξ_1 and ξ_2 in Eqn. (6) are unsigned magnitudes. To apply the sign test in Eqn. (4), we must use the sign of eigenvalue λ_2 from the corresponding gray-scale image.

Experiments and Results

We test our method on a set of color images consisting of photomicrographs [18], nature photos, and satellite imagery [19]. For each such image, a vessel map image is generated that can be used for detection and segmentation of tubular structures, and vessel segmentation and enhancement. The main purpose of the vessel map is to increase the separability of vessel structures from the background. Segmentation can be obtained by thresholding the vessel map, and enhancement can be achieved by pixel-wise multiplication with the input image. Due to the variability in the scale of vessels, the vessel map is constructed using a multi-scale scheme. Five scales of Gaussian are employed for each image, with σ = 1,2,3,4 and 5. Gamma-normalized derivatives are also used with γ = 0.5 as in [1]. The *blobness* and *backgroundness* parameters β and c are set to 0.5. The results are combined across the scales by the maximum rule [1], which is to use the maximum vesselness response across all scales.

Several examples of tests on color images are shown in Figures 1(a)-4(a). The vessel maps are generated based on both the color-Hessian approach and the traditional grayscale-Hessian approach. For ease of comparison, all vessel maps are normalized

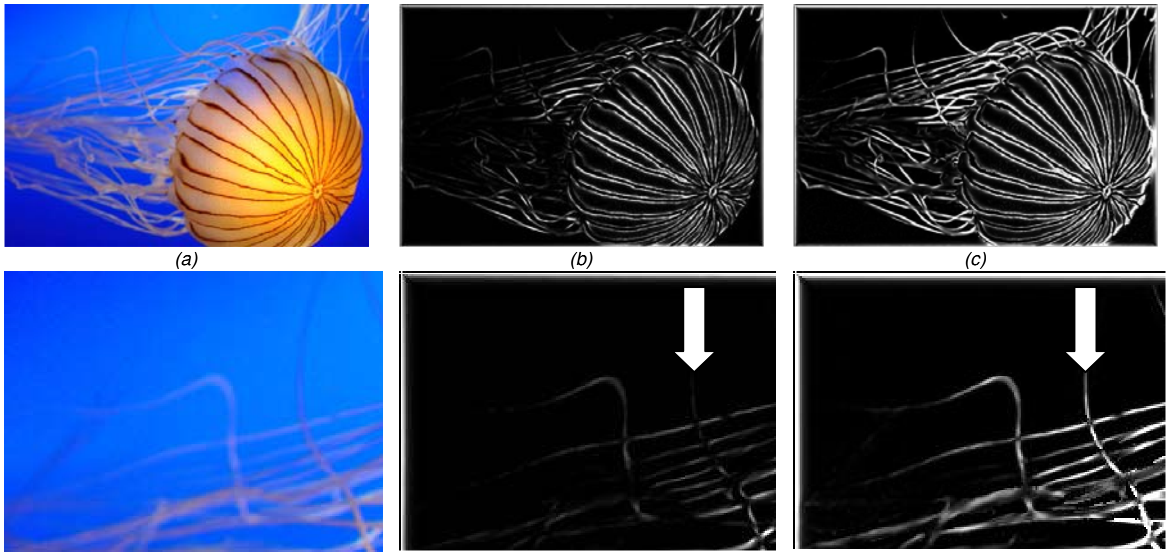


Figure 1. *(a) Photo of a jellyfish [20] (b) Grayscale-based Hessian result in which the tentacles are not detected due to approximate iso-luminance. (c) Color-based Hessian result in which the tentacles are more clearly delineated. (d)-(f) Scaled up version of the top-left corners of (a)-(c) respectively.*

by scaling vesselness intensity to [0,1] and then scaled for better visualization.

Due to the lack of ground truth segmentations paired with the available color image data, we are unable to quantify our results numerically. Nevertheless, the advantage of our method is qualitatively quite clear based on visual inspection of the segmented vessels. As can be seen by comparing Figures 1(c)-4(c) with Figure 1(b)-4(b), the color Hessian achieves better results than the grayscale version in term of the vessel map. In the grayscale-derived vessel maps, there is low vesselness found for vessels that differ in color from the background, but are nonetheless iso-luminant to it. However, even in the regions where luminance of the vessel and background differ, the results of the color Hessian show higher vesselness contrast. Color is an important discriminative property of objects, and the results demonstrate that it provides sufficient extra information to distinguish between background and objects in cases where the traditional luminance-based method fails.

Conclusion

The Hessian matrix can be used to estimate curvature and so provides a good foundation for identifying interesting image features such as tubular vessels and blobs. In generalizing the use of the Hessian from grayscale to color images, however, the problem that arises is the possible information loss caused by cancellation of derivatives in opposing directions from the separate color channels. To overcome this problem, we employ the quaternion representation of color, which encodes an RGB color in a single quaternion number. Information loss is avoided by extracting the eigenvalues from the quaternion-valued Hessian matrix based via QSVD. The quaternion-based method demonstrates improved performance in term of the resulting vessel map, which is important for vessel segmentation and enhancement. Future work will include experiments with 4-channel data (higher dimensions are not possible since quaterions are 4-tuples), and for Hessian-based interest point/blob detection.

References

[1] A. Frangi, W. Niessen, K. Vincken and M. Viergever, Multiscale vessel enhancement filtering, *Proc. of the MICCAI'98 Lecture Notes in Computer Science vol. 1496*, Springer-Verlag, Berlin, pp. 130–137. (1998).

[2] K. Mikolajczyk , C. Schmid, An Affine Invariant Interest Point Detector, *Proceedings of the 7th European Conference on Computer Vision-Part I*, pp..128-142. (2002)

[3] A. P. Condurache, T. Aach, K. Eck, J. Bredno, S. Grzybowsky and H.G. Machens, Vessel Segmentation for Angiographic Enhancement and Analysis, In: *Bildverarbeitung für die Medizin. (Algorithmen, Systeme, Anwendungen),* Heidelberg, pp.173-177. (2005).

[4] H. Bay, T. H. Tuytelaars, L.J. Van Gool, "SURF: Speeded Up Robust Features," *ECCV06*, I, 404-417 (2006).

[5] A. Ming, H. Ma, "A blob detector in color images," *Proceedings of the 6th ACM international conference on Image and video retrieval*, 364-370 (2007).

[6] Y. Du, D.L. Parker, W. Davis, "Vessel Enhancement Filtering in Three-dimensional MR Angiography," *JMRI*, 5, 151-157 (1995).

[7] B.E. Chapman, D.L. Parker. "3D multi-scale vessel enhancement filtering based on curvature measurements: application to time-of-flight MRA," *Med Image Anal*. 9(3),191-208 (2005).

[8] A. P. Condurache, T. Aach, S. Grzybowski, H.-G. Machens, Vessel Segmentation and Analysis in Laboratory Skin Transplant Micro-angiogram, *Proceedings of the Eighteenth IEEE Symposium on Computer-Based Medical Systems*, Dublin, Ireland, pp. 21-26. (2005)

[9] H. Deng, W. Zhang, E. Mortensen, T. Dietterich, Principal Curvature-based Region Detector for Object Recognition, *IEEE Conference on Computer Vision and Pattern Recognition*. (2007).

[10]. K. Mikolajczyk and C. Schmid. "Scale and affine invariant interest point detectors," *IJCV*, 60(1), 63–86 (2004).

[11] C. Harris and M. Stephens, A combined corner and edge detector, *Alvey Vision Conf.*, pp. 147–151. (1988).

[12] K. Mikolajczyk, T. Tuytelaars, C. Schmid, A. Zisserman, J. Matas, F. Schaffalitzky, T. Kadir, and L. V. "Gool. A comparison of affine region detectors," *IJCV*, (2005).

[13] N.L. Bihan, S.J. Sangwine, Quaternion principal component analysis of color images, *ICIP*, pp. 809-812. (2003).

[14] S.C. Pei, J.H. Chang, J.J. Ding, "Quaternion matrix singular value decomposition and its applications for color image processing," *ICIP,* pp. 805-808. (2003).

[15] S. Di Zenzo, "Note: A note on the gradient of a multi-image," *Computer Vision, Graphics, and Image Processing,* 33, 1, 116–125 (1986).

[16] M. Kass and A. Witkin, "Analyzing oriented patterns," *Computer Vision, Graphics, and Image Processing*, 37, 362–385 (1987).

[17] L. Shi, and B. Funt, Quaternion Colour Texture, *AIC'2005 Proc. 10th Congress of the International Color Association,* Granada, (2005).

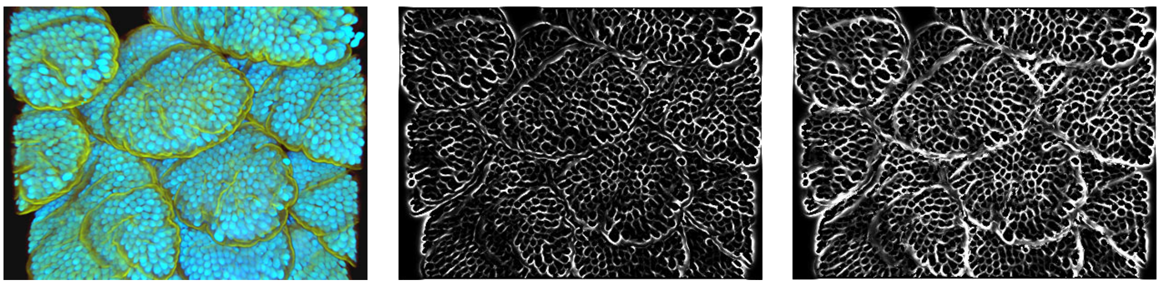
[18] http://www.nikonsmallworld.com (access date: 2008/4/11)

[19] http://geology.com/world-cities/ (access date: 2008/4/11)

[20] http://www.funny-potato.com/jellyfish.html (access date: 2008/4/11)

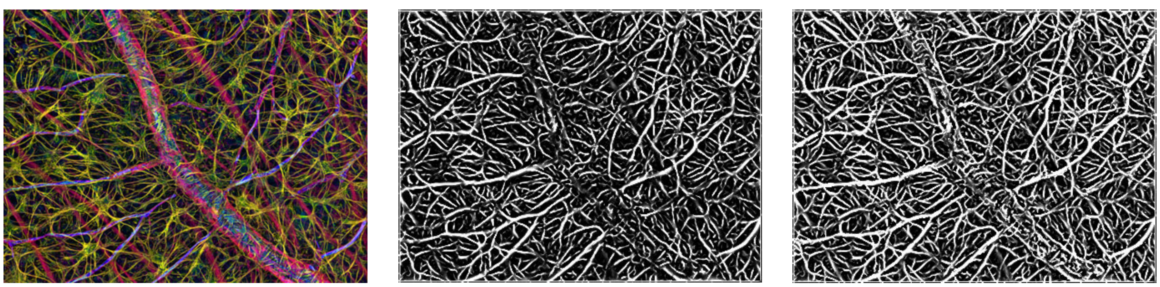
Author Biography

Lilong Shi received his B.Sc. and M.Sc. in Computer Science from Simon Fraser University, Vancouver, Canada, in 2003 and 2005. He is currently a PhD candidate at Simon Fraser University.. His research interests are in illumination estimation, color constancy, and image processing.



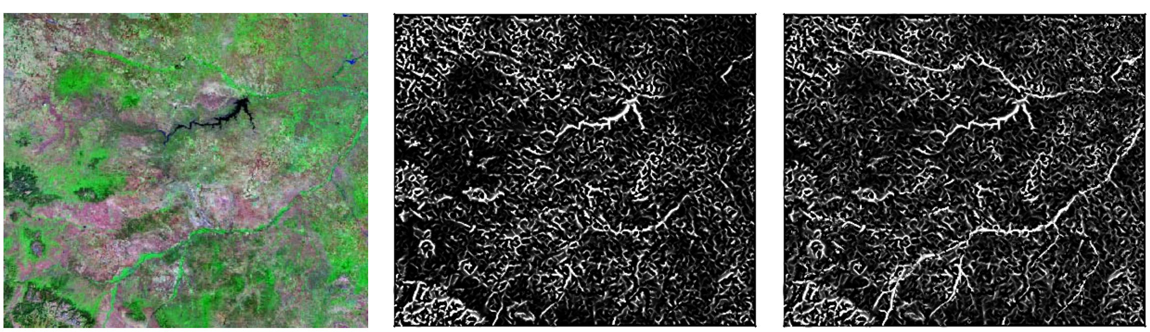
(a) (b) (c)

Figure 2. *(a) A two-photon fluorescence microscopy image of villi of the mouse small intestine [18]; (b) Grayscale-based Hessian result in which the curvature measure around green contours is low because they are similar in intensity to the blue background; (c) Color-based Hessian result in which the green tubular structures are clearly delineated.*



(a) (b) (c)

Figure 3. *(a) Fluorescence and confocal microscopy photo of rat retina astrocytes and blood vessels[18]; (b) Grayscale-based Hessian result where the method fails to detect the dominant vessel across the center line of the image. (c) Color-based Hessian result.*



(a) (b) (c)

Figure 4. *(a) An example satellite image[19]; (b) Grayscale-based Hessian result in which the green vessel-like structure is missed. (c) Color-based Hessian result in which the green vessel-like structure is identified.*