Vessel Crawlers: 3D Physically-based Deformable Organisms for Vasculature Segmentation and Analysis

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Abstract

We present a novel approach to the segmentation and analysis of vasculature from volumetric medical image data. Our method is an adoption and significant extension of deformable organisms, an artificial life framework for medical image analysis that complements classical deformable models with high-level, anatomically-driven control mechanisms. We extend deformable organisms to 3D, model their bodies as tubular spring-mass systems, and equip them with a new repertoire of sensory modules, behavioral routines, and decision making strategies. The result is a new breed of robust deformable organisms, vessel crawlers, that crawl along vasculature in 3D images, accurately segmenting vessel boundaries, detecting and exploring bifurcations, and providing sophisticated, clinically-relevant structural analysis. We validate our method through the segmentation and analysis of vascular structures in both noisy synthetic and real medical image data.

1. Introduction

In modern internal medicine non-invasive imaging procedures are often crucial to the diagnosis of complex and serious cardiovascular, pulmonary, renal, aortic, neurological, and abdominal diseases [2]. Common amongst the diagnosis of these diseases is the use of volumetric angiography to highlight vasculature. Consequently, in order to both efficiently and accurately deal with the voluminous patient data there exists a need for automated filtering, segmentation and analysis techniques. A large volume of work exists on segmenting vasculature. Although, a detailed review of all existing techniques is beyond the context of this paper, we will summarize relevant methods and refer interested readers to [9] for a detailed survey.

Vessel filtering algorithms typically fall into either of two categories: Those that take advantage of the strong edges at vessel walls and diffuse in a way to preserve edges [19, 22], and others that use geometrical features of vessels to describe tubular object preserving filters [4, 10, 16, 17].

Early vascular segmentation techniques were developed based on variants of thresholding and region growing [8, 18]. More recently methods have been developed in most categories of computer vision including pattern recognition, model based approaches, tracking methods, artificial intelligence methods, neural networks and geometrical methods [9]. Our proposed method can be loosely classified as an artificially intelligent deformable model.

Energy minimizing deformable models were applied using both implicitly [21, 14] and explicitly [23, 12, 3] defined contours or surfaces. The more robust of these vessel segmentation methods use tubular geometrical features to drive the segmentation process [12, 23, 3]. However, common amongst these works is a search for maximal responses across a user-defined range of scales, which is prone to problems where noise is prevalent (section 2.1). Clearly, if vascular scale at each voxel was known apriori then the search would be unnecessary and the filtering results optimal. One related method that attempts to locally resolve the scale issue was proposed by Aylward and Bullitt, who define a radial kernel to detect local scale, as their method follows intensity ridges in 3D given multiple seed points [1]. As this method is the most similar to ours, that we know of, we provide comparison to their enabled forms of analysis and quantitative validation (sections 2.4 and 3.1.1).

Bottom-up, pixel-driven segmentation techniques, however, are greatly limited by noise as they lack higher-level processes capable of incorporating and accumulating structural knowledge and a global view of the vasculature. Furthermore, even upon success, these methods often require complex labelling procedures to analyze the resulting segmentation. To the best of our knowledge, no single robust tubular-geometry-driven method exists that integrates top-down and bottom-up control strategies, provides simultaneous segmentation and analysis of vasculature at the locally optimal scale, and avoids the setting of assumed globally-optimal low-level parameters to drive the minimization of a single fixed energy functional. Our approach meets these
criteria while addressing the limitations of current state-of-the-art techniques. Furthermore, given the clinical application, experts are provided with the necessary means to interact with and control the segmentation and analysis process where desired. This interaction enables medical experts to focus on areas of interest within large volumes, ignore unimportant branch points, and easily repair erroneous segmentations whereas upon failure other methods would require the tweaking of low-level parameters.

Specifically, our method does not rely on the minimization of a single energy functional, but rather minimizes a variety of energy functionals at various locations and times, based on dynamically-chosen behavioral methods and automatically-detected locally-optimal filtering parameters. To this end, we adopt the artificial life (AL) based approach to medical image analysis, deformable organism [20, 6], extend it to 3D, and equip it with a tubular approach to medical image analysis, deformable or- ganism [20, 6], extend it to 3D, and equip it with a tubu-

lar geometry, physically-based locomotion capabilities, and vasculature-specific: sensory modules, behavioural rou-
tines, and decision-making strategies (Figure 1). The re-

sult is a robust technique for the segmentation and analysis of vasculature in medical images. We begin with an overview of the geometrical properties of tubular structures, then de-
scribe an overview of the vessel crawler, followed by details of its segmentation and analysis processes.

2. Methods

In this section we provide an overview of the vessel crawler and its use in the segmentation and analysis of vas-
culature in medical images. We begin with an overview of the geometrical properties of tubular structures, then de-
scribe an overview of the vessel crawler, followed by details of its segmentation and analysis processes.

2.1. Geometrical Properties of Tubular Structures

The eigenvalues, $|\lambda_1| \leq |\lambda_2| \leq |\lambda_3|$, of the Hessian, $H_\sigma$, computed at scale $\sigma$ defined as $H(\mathbf{I} * G_\sigma)(x)$, describe the principle curvatures at $x$ [13]. Specifically, the smallest eigenvalued eigenvector $\mathbf{v}_1$ will point along the vessel, while $\mathbf{v}_2, \mathbf{v}_3$ will be orthogonal (Figure 4). Ratios between the eigenvalues give way to Frangi et al.’s vesselness mea-
sure at a single scale

$$V_\sigma(x) = \begin{cases} 0 & \text{if } \lambda_2 > 0 \text{ or } \lambda_3 > 0 \\ (1 - e^{-\frac{R_A^2}{\sigma^2}}) e^{-\frac{R_B^2}{\sigma^2}} (1 - e^{-\frac{R_C^2}{\sigma^2}}) & \text{o/w} \end{cases}$$

in which $R_A$ differentiates between plates and lines, $R_B$ measures deviation from blob like structures, and $S$ emphasizes areas of high contrast [4]. Frangi et al. suggest setting $\alpha$ and $\beta$ to 0.5 and describe the optimal value for $c$ to be the maximum norm of the Hessian. However, images typically contain vasculature of varying radii requiring Frangi et al. to define their final vesselness measure as the maximum re-
sponse of equation 1 across a range of scales. Furthermore, they propose a fixed value for $c$, where clearly as narrowing vessels dim from low blood flow the maximal norm of the Hessian within those vessels will change as well. To rectify this we derive locally optimal values for both $\sigma$ and $c$ (section 2.3), thereby, greatly enhancing the robustness of the method to image noise by focusing on local vessels of the correct radius and intensity levels (Figure 2).

2.2. Vessel Crawler Overview

Our vessel crawler was built under a multilevel AL mod-
delling approach consisting of four primary layers: cognitive, behavioral, physical, and geometrical. Specifically, the cognitive layer makes decisions based on the vessel crawler’s current state, anatomical knowledge, and its sur-
rounding environment (the image volume including the vas-
culature). Decisions could be made to sense information, to
deform based on sensory data, or to spawn a new organism upon the detection of a bifurcation. All of these actions are described under the behavioral layer of the organism, and they rely upon both the physical and geometrical layers for implementation. For example, the act of moving towards a sensed target location is described by the ‘growing’ behavioral method. The cognitive center gathers sensory input using the ‘sense-to-grow’ sensory module, decides the correct location via the ‘where-to-grow’ decision module, elicits the act of ‘growing’, and then conforms to the vascular walls by ‘fitting’. In turn, each of these methods relies upon the physical and geometrical layers to carry out tasks, such as maintaining model stability through the application of stabilization springs. Consequently, we have a framework with many independent layers of abstraction, each built upon the implementation of independent modules and or processes. We provided details in section 2.3.

2.3. Segmentation

DefOrgs embody a modular framework, where interchangeable instances of primary modules (geometric, deformation, sensor, decision, behavior) are systemically designed and interchanged to adapt to the specific problem (Figure 1). Our vessel crawler employs a tubular geometrical module, a physically-based deformation module, three sensory modules, numerous decision modules, and three primary behavioral modules; each of which will be briefly explained here.

The tubular geometric module is parameterized by both section length (distance between neighboring medial masses), and radial resolution (proportional to the number of circumferential boundary masses) (Figure 3). This layered medial shape representation enables intuitive deformations [5], wherein the medial axis governs the bending and stretching of the vessel crawler and is connected to its boundary nodes to control thickness.

The Newtonian physically-based deformation module in [7] is extended to 3D. Our external forces include forces derived from the volumetric image gradient and a drag force, while internal forces are supplied through Hooke’s law and dampening spring forces.

The sensory modules are driven by the vessel crawler’s decisions, wherein optimal parameters are derived through basic sensory modules (local image intensity, standard deviation, and vessel scale) and then passed to more advanced sensory modules (projective spherical and Hessian-based). As noted in section 2.1 the Hessian and its associated filters must be computed at locally optimal scale. Our innovative approach to this problem obtains the locally optimal scale from the radius of the leading (front-most) layer of the vessel crawler after it has deformed to the local vessel. To this end we have developed two main sensory modules that feed into the final ‘sense-to-grow’ module.

Projective spherical: Captures volume intensity information on the surface of a hemisphere centered around the vessel crawler and normal to its front-most layer (Figure 4). A locally optimal connected-components filter is used to ensure only relevant information to the crawler is present on the surface. Let \( V_\sigma(x) \) be the output of equation 1 with \( \sigma \) equal to the current radius, and \( c \) the maximum norm of \( H_\sigma \) computed for all voxels within the contour of the vessel at the previous layer. Consequently, filtering is performed with locally optimal parameters instead of searching across \( \sigma \) and assuming a globally optimal \( c \). The filter returns all voxels with intensities and vesselness measurements that lie within ranges determined using maximum-likelihood-estimates of Gaussian distributions sampled from the previously segmented region of the vessel. We motivate the use of these filters in section 3.1.1. Finally, intensities are projected onto a plane normal to the hemisphere via conversion from spherical to cartesian co-ordinates, to reduce the dimensionality of the sensor. The module then returns a grow direction \( \vec{v}_s \) for each circular region located on the plane, where eccentricity is used to measure the circularity. Specifically, only those regions whose eccentricity matches that of the local vessel within some user set tolerance are accepted. This module is used for crawling along the vessel, bifurcation detection, and crawler termination.

Hessian-based: Is employed for initial vessel alignment, and is provided to help regularize the grow direc-
Figure 3. Vessel crawler’s geometrical and physical construction. From left to right: Masses, circumferential stability springs across sequential layers, scaled up version of boxed region, radial springs in boxed region.

Figure 4. A vessel crawler (gray) utilizing a hemispherical off-board sensor with output (left). Primary and secondary eigenvectors of the Hessian computed at optimal scale (top right).

...tion, \( \vec{v}_s \), obtained from the projective spherical sensory module. Again, using locally optimal scale, the local vessel direction \( \vec{v}_1 \) can be obtained from the eigenvectors of the Hessian. Since the direction is ambiguous, we set \( \vec{v}_h = \text{sign}(\vec{v}_s \cdot \vec{v}_1)\vec{v}_1 \) [1].

**Sense-to-grow:** Returns the final grow direction as an average of the previous two sensory modules. Given \( \vec{v}_s \) from the projective spherical sensory module, and \( \vec{v}_h \) from the Hessian-based sensory module the final direction is taken as:

\[
\vec{v}_f = \frac{\vec{v}_s + \vec{v}_h}{\|\vec{v}_s + \vec{v}_h\|}
\] (2)

The vessel crawler can make a number of key **decisions** at any point in its life span, where each decision can be based on sensory input, anatomical knowledge, or user interaction. It should be noted that through our unique framework the user is able to override any of these key decision functions at any time during the organism’s life cycle, and hence can illicit intuitive real-time control over the segmentation process. Namely, the vessel crawler can decide: where to grow to next, the validity of hypothesized bifurcations, or to terminate its execution.

**Where-to-grow:** The organism must dynamically decide where to grow to at each point of its execution cycle. This decision is based on input from the ‘sense-to-grow’ module as explained above.

**Bifurcation verification:** When the ‘Projective spherical’ sensory module returns a possible bifurcation, indicated by an extra circular region, the vessel crawler must make a decision as to whether the newly detected region is a tangent vessel, image noise, or a valid bifurcation. When a potential bifurcation occurs it is clear from figure 4 that two highly circular intensity regions will appear on the hemispherical slice instead of one. The verification process consists of measuring intensities along rays projected from the current medial node to each proposed bifurcation. Specifically, end point intensity and standard deviation are measured and compared to the same maximum-likelihood-estimates used in the ‘projective spherical’ sensory module. A bifurcation is accepted if the ray’s intensity lies within 2 standard deviations of the estimated mean intensity and its standard deviation is less then or equal to the estimate.

**Terminate:** Each vessel crawler acts as an autonomous agent, and as such requires the capability of deciding upon its own termination. Our organism can terminate on any one of the following three conditions, many of which are based on anatomically driven criteria: layer eccentricity, failure to grow, or upon reaching its destination.

Vessels cross-sections typically appear circular, therefore, we use eccentricity to measure the validity of the crawler’s cross-section. Eccentricity for layer \( l \) is expressed as

\[
\text{ecc}(l) = \lambda_{\text{max}} / \lambda_{\text{min}}
\] (3)

where \( \lambda_{\text{max}} \), and \( \lambda_{\text{min}} \) denote the largest and smallest eigenvalues of the Euclidean distance covariance matrix calculated across the set of boundary nodes.

Growth failure occurs when the ‘sense-to-grow’ module sets a failure to grow flag. This can be caused by a variety of reasons including attempting to grow outside the volume boundaries, or encountering a vessel-less region of the volume. Simply put, when a vessel terminates the local intensities will no longer satisfy the tubular geometrical constraints enforced by equation 1, and hence the spherical sensory module will return an empty value.

An organism is deemed to have reached its destination when the Euclidean distance between the medial node...
of its newest layer, and its destination is less than the current medial growth step-size.

Each of the vessel crawlers key decisions results in the execution of the appropriate behavior using the concluded locally optimal parameters such as scale, estimated vessel mean and variance, etc. The primary behaviors available to the organism are to grow, to fit the vessel wall, and to spawn new organisms.

Growing: As the vessel crawler grows each new layer must be created and subsequently connected to the current end most layer (Figure 3). The newest layer is aligned to the sensed direction vector, \( v_f \), and then rotated axially to prevent mesh twisting. Once connected the model can be fit to the image data.

Fitting: Fitting is accomplished using 3D image gradient driven deformations simulated by the physics layer. Connections to the previous layer provide smoothness, while stiffer circumferential springs provide local stability to noise, and flexible radial springs allow deformation to the vessel boundary.

Spawning new organisms: In order to explore verified bifurcations the organism must be able to spawn new vessel crawlers. Each spawned vessel crawler is initialized based on the optimal parameters detected by the parent, where the radius is based on the estimated radius of the vessel cross-section detected by the spherical sensory module 4.

2.4. Analysis Enabled

As was previously mentioned, the advantage of DefOrgs in addition to their robust segmentation is their ability to perform intuitive analysis and labelling of the target structure. Available forms of analysis are compared to leading methods in table 1.

![Figure 5](image)

Figure 5. Left: Directed acyclic graph (in black) shown in 3D context overlaid on a plot of the vessels with color corresponding to local radial thickness. Right: Tree showing vascular hierarchy displayed out of context.

3. Results

We provide both qualitative and quantitative validation through numerous synthetic and real examples. We demonstrate the vessel crawlers ability to handle complex topologies, high curvature bendings and windings, significant changes in radius, tangent vessels, and exceedingly noisy synthetic data. We also provide quantitative results on one Computed Tomography Angiography (CTA) vascular phantom and qualitative results on a magnetic resonance angiograph (MRA).

3.1. Synthetic

Our synthetic data serves as a way to quantitatively validate the vessel crawler’s ability to track vessels, latch to vessel walls, and detect bifurcations.

3.1.1 Vessel Tracking

We validate our algorithm on highly curved synthetic vessels (mean intensity 100) containing increasing amounts of Gaussian noise (Figure 7), and motivate the use of each of the connected-component filters used in the sensory modules. As shown in figure 8 purely intensity based methods are ill-equipped to deal with nominal amounts of noise. However, the incorporation of tubular geometrical constraints using optimal scale parameters enables the vessel crawler to track vessels despite noise levels well exceeding those found in clinical data. It is important to mention that the average distance from the closest correct centerline did not exceed 0.6 voxels across all tested noise levels, and the Hausdorff distance from correct centerline remained under 1.5 voxels. Together, these points indicate that the crawler correctly tracked the noisy synthetic vessels and promptly terminated execution where tracking was unreliable, as crawling far off the vessel would greatly increase both values. Furthermore, as [1] also computes the Hessian at locally detected scale, we provide a comparison to their reported results in table 2.
Table 1. Comparison of enabled forms of analysis, where implicit denotes those directly obtained by the method during the segmentation process and PPR denotes those requiring post-processing. Binary methods denote vessel segmentation methods whose output is a binary volume (e.g. level set, region growing).

<table>
<thead>
<tr>
<th>Analysis</th>
<th>Our Method</th>
<th>Aylward [1]</th>
<th>Binary Methods</th>
</tr>
</thead>
<tbody>
<tr>
<td>Branch point detection</td>
<td>Implicit via Binary Verification decision module and shown in figure 5</td>
<td>PPR</td>
<td>PPR</td>
</tr>
<tr>
<td>Cross sectional radii and slices</td>
<td>Implicit via length of radial springs (figure 6)</td>
<td>Implicit</td>
<td>PPR</td>
</tr>
<tr>
<td>Volumes of branches</td>
<td>Implicit via segmentation</td>
<td>Implicit</td>
<td>PPR</td>
</tr>
<tr>
<td>Labelling branch hierarchy - tree graph</td>
<td>Implicit via Binary Verification decision module, and shown in figure 5</td>
<td>PPR</td>
<td>PPR</td>
</tr>
<tr>
<td>Branching angles</td>
<td>Angle between medial vectors of parent and child crawlers</td>
<td>PPR</td>
<td>PPR</td>
</tr>
<tr>
<td>Segment lengths</td>
<td>Sum of distances between medial points</td>
<td>Implicit</td>
<td>PPR</td>
</tr>
<tr>
<td>Label the segments/branches/generations</td>
<td>Implicit via Binary Verification decision module</td>
<td>PPR</td>
<td>PPR</td>
</tr>
<tr>
<td>Distance metric (DM)</td>
<td>Directly calculated from medial node positions, see [2]</td>
<td>Implicit</td>
<td>PPR</td>
</tr>
<tr>
<td>The sum of angles metric (SOAM)</td>
<td>Directly calculated from medial node positions, see [2]</td>
<td>Implicit</td>
<td>PPR</td>
</tr>
<tr>
<td>Inflection count metric (ICM)</td>
<td>Directly calculated from medial node positions, see [2]</td>
<td>Implicit</td>
<td>PPR</td>
</tr>
<tr>
<td>Identify affected regions if problem occurs at a particular location</td>
<td>Highlight all child-vessels of the affected parent</td>
<td>PPR</td>
<td>PPR</td>
</tr>
<tr>
<td>Identify vascular paths to specific locations</td>
<td>Highlight all parent vessels in the vessel tree</td>
<td>PPR</td>
<td>PPR</td>
</tr>
<tr>
<td>Camera paths</td>
<td>Computed from sequential medial nodes (Figure 6).</td>
<td>PPR</td>
<td>PPR</td>
</tr>
</tbody>
</table>

![Figure 7. Example Synthetic data. Left: Surface rendering of noise-free vessel. Right: Oblique slices depicting two levels of Gaussian noise: $\sigma=25,100$ (mean vessel intensity 100).](image1)

![Figure 8. Results of vessel crawler on synthetic data using image intensity (blue), vesselness (green), and both (red) for the connected components (section 7.7). Fraction of total vessel length segmented (left), mean Hausdorff distance (in voxels) to correct vessel centerline (middle) and to boundary (right) normalized by crawler length. All plots are versus standard deviation of additive Gaussian noise with vessel intensity 100.](image2)

3.1.2 Vessel Walls

Figure 8 shows the vessel crawlers ability to obtain correct boundary node placement (segmentation) even under exceedingly noisy situations. Here again the advantages of a locally optimal vesselness measure are clearly visible. The low error shows how robust initialization (obtained by the sensory modules) can allow gradient based deformations to converge to optimal solutions despite significant amounts of image noise, thus avoiding their traditional local-minima-based pitfalls. Here the mean distance from the closest correct vessel boundary did not exceed 0.7 voxels across all tested noise levels.

3.1.3 Bifurcation Detection

We ran our bifurcation detection algorithm on both our own synthetic data and data sets obtained from [11]. The vessel crawler proved capable of detecting bifurcations of varying radii and difficulty in noise-free situations by correctly segmenting all of the Y-junction data provided. For our own synthetic data we tried increasing amounts of noise using a similar Y-junction, showing that the vessel crawler can detect bifurcations along main vessels (radius $\geq 3$ voxels) under highly noisy conditions, and detect minor vessels (ra-
\[ \text{true/false positives for bifurcation detection} \]
\[
\begin{array}{cccccccccccc}
0.5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
\end{array}
\]

Table 3. Vessel radius (rows) versus standard deviation (columns) of additive Gaussian noise. Results in each cell shown as true/false positives.

### 3.2. Medical Images

We provide quantitative results on one CTA vascular phantom, and qualitative results on one human MRA. The vascular phantom, courtesy of [15], is a plexiglass box with silicon gel and nylon tubing with a scan resolution of 0.6 × 0.6 × 1.25 \text{mm}^3 and an average vessel radius of approximately 1.2 mm, while the patient data is an isotropic 416×512×512 MRA with 0.412 \text{mm}^3 voxel size, and 8 bit precision. Results are shown in figures 5, 9, and 10. For the phantom data the crawler correctly detects all branch points, and comparing the vessel crawlers radius measurement to the provided ground truth gives a mean voxel error of 0.287 with a standard deviation of 0.1565. For the MRA the vessel crawler was initialized using a single seed point and initial direction for each of the three root-most vessels. As figure 10 shows, it was able to detect and track the majority of connected vessel segments. In regions where the bifurcation detection algorithm fails, the organism becomes limited to the requirement of multiple seed points similar to that of other methods [1, 3]. However, given the vessel crawler’s use of optimally detected scale, its tracking performance in these situations is at least as good as other Hessian-based approaches, since their maximal filter response includes vesselness measurements affected by noise from non-optimal scales (Figure 2).

### 4. Conclusions

We have developed an improved technique for the segmentation and analysis of branching vasculature in medical images, while extending existing 2D DefOrgs to include 3D physics based deformations, and new behavioral and sensory capabilities. Our crawlers consolidate vessel segmentation and analysis in a single framework in contrast to how other methods typically require segmentation, skeletonization, pruning, branch-point labelling, and then analysis.

Furthermore, the framework requires minimal seed point selection and is amenable to intuitive user-interaction through its physically-based deformations. From our experience thus far the framework seems intuitive for clinical experts to both understand at a high level and control the segmentation and analysis process by guiding the crawler through erroneous regions, adding missing bifurcations, and repairing incorrectly segmented boundaries, without needing to stop, adjust low-level parameters, and run again.

We have demonstrated how our framework overcomes specific problems faced by current vascular segmentation algorithms [2, 3] by detecting bifurcations, deriving locally optimal parameters, and providing simultaneous clinically relevant analysis. For the first time [4]’s filtering technique is performed using locally optimal scale, greatly increasing the filter’s ability to handle noise (Figure 2).
comparison to region growers (RG) the vessel crawlers possess prior-shape knowledge (vs. amorphous shapes in RG), perform highly controlled deformations (vs. selectively adding nearby pixels), allow for intuitive interaction, and readily provide analysis (Table 1). In comparison to deformable models [3, 14]: The latter do not detect branches, require multi-point initialization in every vessel segment, do not know where to start/stop, require error-inducing post-processing for analysis, rely on unintuitive parameter tuning, are not designed to handle complex model fitting strategies, are restricted to a single global energy functional, lack the ability for high-level user-interaction, and do not incorporate locally optimal scale [3].

Our future work will focus on designing behaviors to flag potential aneurysms and stenoses in angiography, and extending the work to other tubular anatomical structures.

5. Acknowledgements

We would like to thank Vincent Luboz et al. for providing us with the phantom CTA scan along with ground truth radial measurements used in [15], Stephen Aylward et al. for providing access to their synthetic data and ground truth measurements used in [1], Lisa Tang for running tests on various data sets, and Liu Shuo for assisting with code development.

References


