Mesh Segmentation via Recursive and Visually Salient Spectral Cuts

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Organization
- Background
- Algorithm overview & contribution
- Nyström method and sub-sampling
- 1D spectral embedding
- Recursive salience-based cut
- Experimental results
- Future work

Segmentation Categories
- Patch type
- Part type

[A. Shamir, 2004]
Related Work

volume based

Skeletonization

Shape descriptor

watershed

scissoring

fuzzy clustering

k-means

surface based

Collision detection

Polarization theorem [Brand & Huang 03]

Classical Spectral Clustering

Compute Distance matrix and transform to Affinity matrix: $A \in \mathbb{R}^{N \times N}$

Find $k$-dim. embedding

$U = [e_1, e_2, \ldots, e_k]$

Eigenvector Decomposition: $A = E \Lambda E^T$

$k$-means clustering on rows of $U$

Problems

Extremely time consuming
- Complexity is at least $O(n^2)$ for pairwise distances.
- Eigenvector decomposition is also expensive even if only leading eigenvectors are needed.

Do not know the appropriate embedding dimension, $k$

Shortcomings of $k$-means

Various Problems of $k$-means

The embedding might not have clear clusters.

Even if there exist clear clusters, $k$-means might not find them correctly.

Even if $k$-means finds them, these clusters might not correspond to visually meaningful mesh segmentations.
Algorithm Overview & Contribution

Contributions

- **Efficiency**
  - Sub-sampling and Nyström method
  - Complexity is $O(pn \log n)$, $p \ll n$, $p$ is iteration count.

- **Visually meaningful segmentation**
  - Based on minima rule
  - Saliency measures: size, protrusion, cut strength

- **Robustness**
  - Recursive 2-way cut
  - Robust sampling scheme

Face Distances

- **Adjacent faces:** [S. Katz and A. Tal, 03]
  \[ d = \delta (\text{geodesic distance}) + (1-\delta) (\text{angle distance}) \]

- **Non-adjacent faces:**
  \[ d = \text{Shortest graph distance in the dual graph} \]
  \[ \delta = \begin{cases} 0 & \text{angle distance} \\ 0.1 & \text{combined distance} \\ 1 & \text{geodesic distance} \end{cases} \]
More on Distances

- Angle distance -> geodesic distance
- Concave angle distance -> convex angle distance

Due to possible noisy dihedral angles, angle distances are histogram-equalized to enhance results.

Nyström Method and Sub-sampling

Nyström Method

- Nyström method extrapolates eigenvectors using sampled rows of a matrix. [C. Fowlkes et al. 2004.]

Sub-sampling

- Only sample two faces.
- Requirements: two sample faces reside on two different parts.
Difficulties

- Segmentation result not known yet.
- Sampling two faces furthest away does not work.
  - Concave region might be bridged by flat regions.

Shape Context

- $shape\_context(\triangle) = sc(\triangle) = (c_1, c_2, \ldots, c_i, \ldots, c_k)$
  - sum up the angle distance along the path
  - geodesic path
  - reference faces

- Reference faces are chosen approximately uniformly.

Sampling using Shape Context

- First sample
  - $f_s = \arg \max_i \| sc(i) \|$
  - Explanation: $f_s$ most likely resides on a peripheral part separated by concavity.

- Second sample
  - $f_r = \arg \max_i \| sc(i) - sc(s) \|$
  - Explanation: $f_r$ most likely resides on a different part.
Distance to Affinity

- Exponential kernel:
  \[ A_{ij} = e^{-\frac{D_{ij}}{\sigma^2}} \]

- Kernel width \( \sigma \) is the average of all distances available.

1D Spectral Embedding \( \vec{Z} \)

- Eigenvectors of \( N \) NOT needed explicitly

- \( \vec{Z} \) is the component-wise ratio of the second and first largest eigenvectors of \( N \).

- \( \vec{Z} \) happens to be the second smallest eigenvector of normalized Laplacian \( L = I - D^{-1}A \), which is used for image segmentation in “normalized cut” [J. Shi and J. Malik, 1997].

Normalized Affinity Matrix

- Instead of \( A \), normalized affinity matrix is used.
  \[ N = R^{-1/2}AR^{-1/2} \]
  where \( R \) is a diagonal matrix, whose diagonals are row sums of \( A \).

- Problem: without knowing all entries of \( A \), \( R \) is unknown, neither is \( N \).

- \( A = \begin{bmatrix} \text{not needed} & \text{needed} \end{bmatrix} \Rightarrow N \)

Problems Solved

- Complexity is dramatically reduced.
  - distance computation
  - eigenvalue decomposition

\[ O(n^2) \rightarrow O(pn \log n), \quad p \ll n \]

\( p \) is the iteration count.
Recursive Salience-based Cut

Incorporate Salience

At each position, the salience of the cut is computed. The most salient cut is chosen to segment the mesh into two parts.

Part boundary is an edge sequence.

Part Salience

\[ \text{salience} = \alpha \cdot \text{size} + \beta \cdot \text{cut strength} + \gamma \cdot \text{protrusiveness} \]

[ D. Hoffman and M. Singh, 1997; D. Page, 2003 ]

Salience is always measured on the smaller part.

Measurement

After each face is added into the part, all the measures can be updated in constant time.
Iteration

- Decomposed sub meshes are pre-segmented to compute saliencies and inserted into a max-heap based on salience.
- Next part to segment is always the one with biggest salience.

Stopping Criteria

- User specified part number is reached.
- No part is more salient than the user given threshold.

Problems Solved

- Needless to know the number of embedding dimension.
- Bypass the shortcomings of k-means.

Experimental Results
Sample Segmentations

<table>
<thead>
<tr>
<th>Model (# faces)</th>
<th>Initialization</th>
<th>Sampling</th>
<th>Embedding</th>
<th>Line Search</th>
<th>Total</th>
<th># parts</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heart (16K)</td>
<td>0.04</td>
<td>0.03</td>
<td>0.01</td>
<td>0.09</td>
<td>0.17</td>
<td>5</td>
</tr>
<tr>
<td>Dolphin (2K)</td>
<td>0.03</td>
<td>0.14</td>
<td>0.03</td>
<td>0.25</td>
<td>0.45</td>
<td>8</td>
</tr>
<tr>
<td>Bird (3K)</td>
<td>0.07</td>
<td>0.15</td>
<td>0.02</td>
<td>0.24</td>
<td>0.48</td>
<td>5</td>
</tr>
<tr>
<td>Doe-pet (4K)</td>
<td>0.08</td>
<td>0.45</td>
<td>0.15</td>
<td>0.79</td>
<td>1.47</td>
<td>29</td>
</tr>
<tr>
<td>“Snake” (12K)</td>
<td>0.24</td>
<td>0.40</td>
<td>0.12</td>
<td>0.61</td>
<td>1.37</td>
<td>4</td>
</tr>
<tr>
<td>Bowl (13K)</td>
<td>0.25</td>
<td>0.30</td>
<td>0.07</td>
<td>0.32</td>
<td>0.94</td>
<td>3</td>
</tr>
<tr>
<td>Machine part (20K)</td>
<td>0.46</td>
<td>1.82</td>
<td>0.47</td>
<td>2.1</td>
<td>4.85</td>
<td>6</td>
</tr>
<tr>
<td>Horse (40K)</td>
<td>0.54</td>
<td>5.3</td>
<td>1.79</td>
<td>5.13</td>
<td>12.76</td>
<td>19</td>
</tr>
<tr>
<td>Bunny (70K)</td>
<td>1.02</td>
<td>7.7</td>
<td>1.86</td>
<td>7.29</td>
<td>17.37</td>
<td>14</td>
</tr>
<tr>
<td>Igu (200K)</td>
<td>2.92</td>
<td>19.7</td>
<td>4.54</td>
<td>13.48</td>
<td>40.64</td>
<td>5</td>
</tr>
</tbody>
</table>

Timing in seconds
- Xeon 2.2 GHz, 1GB RAM
- Without sampling: 30s for 4000-face mesh, 2.8 GHz [R. Liu and H. Zhang 04].

Comparison vs k-means

- Automate the whole process
  - Automatic parameters setting
  - Robust stopping criteria

- Improve salience measure
  - More accurate salience factor measure
  - Better combination of salience factors

- Robustness
  - Scale invariance
  - Connectivity free
Appendix – Nyström Approximation

\[ A^{n \times n} = \begin{bmatrix} X & Y \\ Y & Z \end{bmatrix} \]

What are \( A \)'s eigenvectors \( \vec{U} \)?

\[ X = U \Lambda U' \Rightarrow \vec{U} = \begin{bmatrix} U \\ Y U \Lambda^{-1} \end{bmatrix} \]

Only need to know \( A^{n \times n} = \begin{bmatrix} X & k \times k \\ Y \end{bmatrix} \).

Appendix – Salience Factors

- \( \text{size} = \frac{\text{area}(Q)}{\text{area}(M)} \)

- \( \text{protrusiveness} = 1 - \frac{\text{base area subtended by } \partial Q}{\text{area}(Q)} \)

\( = 1 - \frac{4 \lambda_1 \lambda_2}{\text{area}(Q)} \) where \( \lambda_1, \lambda_2 \) are the leading eigenvalues of the

covariance matrix for the vertices along \( \partial Q \).

- \( \text{cut_strength} = \frac{1}{m} \sum_{e \in \partial Q} \frac{\text{angle_distance}(e)}{\max_{\partial Q} \text{angle_distance}(M)} \)

\( \text{angle_distance}(e) = \) Angle distance of the two faces having common edge \( e \).

Appendix - Surface Pre-smoothing

- Noisy surface are slightly smoothed with several iterations of Laplacian smoothing.
Appendix - Post Boundary Smoothing

- The direct resulting boundaries generally are already smooth.

- Only some local artifacts are smoothed using a simple morphological smoothing.