Original slides by Dennis Zorin
(with minor modifications)
Classification

Classification criteria

- stationary or nonstationary
- type of refinement rule (primal or dual)
- type of mesh (triangular or quad or...)
- approximating or interpolating
Refinement Rules

- Primal (face refinement)
- Dual (vertex refinement)

Subdivision for Modeling and Animation
Dual scheme for triangle mesh

Refer to assignment problem on midpoint subdivision.
Refinement Rules

- Primal faces form a quad tree
- Dual vertices form a quad tree

Subdivision for Modeling and Animation
Approximation and Interpolation

Advantages

- approximating schemes
- based on splines, small support
- interpolating schemes
- control points on surface
- in-place implementation

- No additional storage other than those needed to store control points generated along subdivision.

- Consider approximating, new value for vertex may need to be stored elsewhere since its old value is needed for other computations
Subdivision Schemes

Primal (vertex insertion)

- Approximating: Catmull-Clark, Loop
- Interpolating: Kobbel, Butterfly

Dual (corner cutting)

- Approximating: Doo-Sabin, Midedge
- Interpolating: Dyn-Levin-Liu (non-linear)
Geometric Rules

How many rules we need?
- first, need rules for interior and boundaries
- also need corners
- creases come for free with boundary rules
Boundaries

Three types of points:

- Smooth
- Convex corners
- Concave

Important:
need separate rules for concave and convex!
Vertex Types

Four basic types

- interior
- boundary smooth
- convex
- concave
Boundaries and Creases

- If we know how to do boundaries, we (almost) know how to do creases
- Boundary independent of the interior

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Crease Examples

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Subdivision Schemes

- **Primal (vertex insertion)**
  - Approximating: Catmull-Clark
  - Interpolating: Kobbelt

- **Dual (corner cutting)**
  - Approximating: Doo-Sabin, Midedge
  - Interpolating: Butterfly

Subdivision for Modeling and Animation
Loop Scheme

Primal, triangular meshes, approximating

- Two rules for interior:

vertex

edge

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Loop Scheme

Regular masks
- derived from three-directional quartic box spline

Subdivision for Modeling and Animation
Loop Scheme

Extraordinary vertex masks

- Original Loop:
  \[ \beta = \frac{1}{K} \left( \frac{5}{8} - \left( \frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{K} \right)^2 \right) \]

- Warren, for \( K > 3 \)
  \[ \beta = \frac{3}{8K} \]
Loop Scheme: boundaries

- For **smooth** boundaries or edges tagged as crease edges, use special rules to produce **cubic spline curve** along boundary or crease.

- These rules only depend on points along the boundary or crease.

- For a corner point, use interpolation.

- Concave boundary vertices are harder to deal with.
Subdivision Schemes

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Subdivision for Modeling and Animation
Catmull-Clark Scheme

Primal, quadrilateral, approximating tensor-product bicubic splines

Subdivision for modeling and animation
Reduction to a quadrilateral mesh

Do one step of subdivision with special rules; all polygons become quads.
CATMULL-CLARK SCHEME

Extraordinary vertices

\[ \gamma = \frac{1}{4K} \]

\[ \beta = \frac{3}{2K} \]
Subdivision Schemes

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Subdivision for Modeling and Animation
Butterfly Scheme

Primal, triangular, interpolating

- only one rule
- needs larger support

regular mask
Butterfly Scheme

Extraordinary vertices

- coefficients derived to ensure good eigenvalues and eigenvectors

\[ s_j = \frac{1}{K} \left( \frac{1}{4} + \cos \frac{2j\pi}{K} + \frac{1}{2} \cos \frac{4j\pi}{K} \right) \]

\[ K > 4 \]

\[ K = 3: \ s_0 = 5/12, \ s_{1,2} = -1/12 \]

\[ K = 4: \ s_0 = 3/8, \ s_2 = -1/8, \ s_{1,3} = 0 \]

Take average if both end points are irregular
**Subdivision Schemes**

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*Subdivision for Modeling and Animation*
Subdivision Schemes

- **Primal (vertex insertion)**
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Subdivision for Modeling and Animation
Doo-Sabin Scheme

Dual scheme, quadrilateral
- extends tensor-product biquadratic splines
**Doo-Sabin Scheme**

- after one step, all valences = 4
- rule for extraordinary polygons:
  \[ \alpha_0 = \frac{1 + 5K}{4} \]
  \[ \text{for } i = 1 \ldots K - 1 \]
  \[ \alpha_i = \frac{1}{K} \left( 3 + 2 \cos \frac{2i\pi}{K} \right) \]
MIDEDGE SCHEME

Dual scheme, quadrilateral
- also known as simplest
- extends 4-directional box spline

Subdivision for Modeling and Animation
Summary

- Primal (vertex insertion)
  - Approximating: Catmull-Clark
  - Interpolating: Kobbelt

- Dual (corner cutting)
  - Doo-Sabin, Midedge
  - Loop: Butterfly

Subdivision for Modeling and Animation
Comparison

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Comparison

Loop
Catmull-Clark
Tetrahedron
Butterfly
Doo-Sabin

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Comparison

initial mesh

Loop

Catmull-Clark

Subdivision for Modeling and Animation
Comparison

initial mesh  Loop  Catmull-Clark

Subdivision for Modeling and Animation
Comparison

Butterfly  Catmull-Clark  Loop  Doo-Sabin

Subdivision for Modeling and Animation
Comparison

Subdivision for graphics
- Loop seems to be most reliable
- Catmull-Clark
- Doo-Sabin and other dual schemes may have some computational advantages but surface quality is not so good