SUBDIVISION ZOO

Original slides by Dennis Zorin
(with minor modifications)
Classification

Classification criteria

- stationary or nonstationary
- type of refinement rule (primal or dual)
- type of mesh (triangular or quad or...)
- approximating or interpolating
Refinement Rules

- Primal (face refinement)
- Dual (vertex refinement)
Dual scheme for triangle mesh

Refer to assignment problem on midpoint subdivision.
Refinement Rules

- Primal faces form a quad tree
- Dual vertices form a quad tree
Subdivision Schemes

Primal (vertex insertion)
- Approximating: Catmull-Clark
- Interpolating: Kobbelt

Dual (corner cutting)
- Doo-Sabin, Midedge
- Butterfly

Subdivision for Modeling and Animation
Geometric Rules

How many rules we need?
- first, need rules for interior and boundaries
- also need corners
- creases come for free with boundary rules
Boundaries

Three types of points:

- smooth
- convex corners
- concave

Important: need separate rules for concave and convex!
**Vertex Types**

Four basic types:
- interior
- boundary smooth
- convex
- concave
Subdivision Schemes

- Primal (vertex insertion)
  - Approximating: Catmull-Clark
  - Interpolating: Kobbelt

- Dual (corner cutting)
  - Doo-Sabin, Midedge
  - Loop: Butterfly

Subdivision for Modeling and Animation
Loop Scheme

Primal, triangular meshes, approximating

- Two rules for interior:

vertex

edge

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Loop Scheme

Regular masks
derived from three-directional quartic box spline

Subdivision for Modeling and Animation
Loop Scheme

Extraordinary vertex masks

- original Loop:
  \[ \beta = \frac{1}{K} \left( \frac{5}{8} - \left( \frac{3}{8} + \frac{1}{4} \cos \frac{2\pi}{K} \right)^2 \right) \]

- Warren, for \( K > 3 \)
  \[ \beta = \frac{3}{8K} \]
Loop Scheme: boundaries (aside)

- For **smooth** boundaries or edges tagged as crease edges, use special rules to produce **cubic spline curve** along boundary or crease.

- These rules only depend on points along the boundary or crease.

- For a corner point, use interpolation.

- Concave boundary vertices are harder to deal with.
Subdivision Schemes (aside)

Primal (vertex insertion)

- Approximating: Catmull-Clark
- Interpolating: Kobbelt

Dual (corner cutting)

- Doo-Sabin, Midedge
- Dyn-Levin-Liu (non-linear)

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Catmull-Clark Scheme (aside)

Primal, quadrilateral, approximating tensor-product bicubic splines

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Reduction to a quadrilateral mesh

- do one step of subdivision with special rules; all polygons become quads
CATMULL-CLARK SCHEME (aside)

Extraordinary vertices

\[ \gamma = \frac{1}{4K} \]

\[ \beta = \frac{3}{2K} \]
Subdivision Schemes

- **Primal (vertex insertion)**
  - Approximating: Catmull-Clark
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Subdivision for Modeling and Animation
Butterfly Scheme

Primal, triangular, interpolating
- only one rule
- needs larger support

regular mask
Butterfly Scheme

Extraordinary vertices

- coefficients derived to ensure good eigenvalues and eigenvectors

\[ s_j = \frac{1}{K} \left( \frac{1}{4} + \cos \frac{2j\pi}{K} + \frac{1}{2} \cos \frac{4j\pi}{K} \right) \]

\[ K > 4 \]

\[ s_0 = \frac{5}{12}, s_{1,2} = -\frac{1}{12} \]

\[ K = 3: s_0 = \frac{3}{8}, s_2 = -\frac{1}{8}, s_{1,3} = 0 \]

Take average if both end points are irregular

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S U B D I V I S I O N   S C H E M E S  (a s i d e)

- **Primal (vertex insertion)**
  - Approximating: Catmull-Clark
  - Interpolating: Kobbel

- **Dual (corner cutting)**
  - Doo-Sabin, Midedge
  - Dyn-Levin-Liu (non-linear)

**See Notes**
Subdivision Schemes (aside)

- **Primal (vertex insertion)**
  - Approximating: Catmull-Clark
  - Interpolating: Kobbelt

- **Dual (corner cutting)**
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Doo–Sabin Scheme (aside)

Dual scheme, quadrilateral
- extends tensor-product biquadratic splines
Doo-Sabin Scheme (aside)

- after one step, all valences = 4
- rule for extraordinary polygons:

\[
\alpha_0 = \frac{1 + 5K}{4}
\]

for \( i = 1 \ldots K - 1 \)

\[
\alpha_i = \frac{1}{K} \left( 3 + 2 \cos \frac{2i\pi}{K} \right)
\]

Subdivision for Modeling and Animation
Midedge Scheme (aside)

Dual scheme, quadrilateral
- also known as simplest
- extends 4-directional box spline

Subdivision for Modeling and Animation
**Summary**

- **Primal (vertex insertion)**
  - Approximating: Catmull-Clark
  - Interpolating: Kobbel

- **Dual (corner cutting)**
  - Approximating: Doo-Sabin, Midedge

**Subdivision for Modeling and Animation**
Comparison

Loop
Catmull-Clark
Cube
Butterfly
Doo-Sabin

Subdivision for Modeling and Animation
Comparison

Subdivision for modeling and animation
Comparison

initial mesh  Loop  Catmull-Clark

Subdivision for Modeling and Animation
Comparison

initial mesh  Loop  Catmull-Clark

Subdivision for Modeling and Animation
COMPARISON

Butterfly  Catmull-Clark  Loop  Doo-Sabin