Cooperative Coverage Extension for Relay-Union Networks

Yong Cui, Xiao Ma, Xiuzhen Cheng, Minming Li, Jiangchuan Liu, Tianze Ma, Yihua Guo, and Biao Chen

Abstract—Multi-hop coverage extension can be utilized as a feasible approach to facilitating uncovered users to get Internet service in public area WLANs. In this paper we introduce a Relay-Union Network (RUN), which refers to a public area WLAN in which users often wander in the same area and have the ability to provide data forwarding services for others. We develop a RUN framework to model the cost of providing forwarding services and the utility obtained by gaining services. The objective of the RUN is to maximize the total Quality of Cooperation (QoC) of users among RUN. Two optimal bandwidth allocation schemes are proposed for both free and dynamic bandwidth demand models. To make our scheme more pragmatic, we then consider a more practical scenario in which the bandwidth capacity of the relays and the minimum demand of the clients are bounded. We prove that the problems under both the single relay and the multi-relay scenario are NP-hard. Three heuristic algorithms are proposed to deal with bandwidth allocation and relay-client association. We also propose a distributed signaling protocol and divide the centralized MRMC algorithm into three distributed ones to better adapt for real network environment. Finally, extensive simulations demonstrate that our RUN framework can significantly improve the efficiency of cooperation in the long term.

Index Terms—Coverage extension, relay-union network, bandwidth allocation, association, cooperative.

1 INTRODUCTION

A public area WLAN often has a relatively fixed user group such as students of a classroom building. Sometimes a user does not have Internet access due to limited AP coverage, poor signal strength, or authentication failure. Multi-hop access is a feasible approach to facilitate these users to employ other online users as relays for data forwarding [1]–[5]. In this paper, we propose a “Relay-Union Network (RUN)”, which contains a union of clients that often wander in the same public area and sometimes provide data forwarding for others. In a RUN, at any moment, a fraction of the union members with direct AP connections play the role of relays and offer forwarding services to others. In RUN, service demands of the clients may be different from each other and vary from time to time. To characterize the satisfaction degree of a client on the obtained forwarding service, we employ a utility function as a metric in our model. Since providing forwarding services for others may have negative impact on the relay’s own performance (e.g., battery power, CPU resource), we also consider a relay cost function in our RUN framework. As all the members in a RUN take the role of relay from time to time, all users mutually benefit from each other in the long term.

In this paper, we first propose a Quality of Cooperation (QoC) model that includes a client utility function and a relay cost function, based on which the QoC of a relay and a client can be well defined. Next we point out that the cutoff bandwidth allocation is a crucial issue in our strategy through analyzing the relationship between the bandwidth and the QoC. Optimal bandwidth allocation schemes are then proposed for two bandwidth demand models. Following that we consider a more practical scenario in which the relay’s bandwidth capacity and the client’s minimum bandwidth demand are respectively upper and lower bounded. We then propose three heuristic algorithms, with two targeting on Single-Relay Multi-Client (SRMC) for bandwidth allocation, and the other one targeting on Multi-Relay Multi-Client (MRMC) for both client-relay association and bandwidth allocation. Finally we introduce a distributed dynamic mechanism of MRMC (D^2MRMC), which consists of a distributed signaling protocol and several distributed algorithms to handle the dynamic scenarios of MRMC. Finally, extensive simulation study is performed, and the results indicate that our framework can significantly improve the total QoC of RUN.

The rest of the paper is organized as follows. Section 2 presents the related research. Section 3 introduces the QoC model and the related definitions. The solutions for single-relay multi-client and multi-relay multi-client are investigated in Section 4 and 5, respectively. The distributed MRMC mechanism and the comparison between MRMC and D^2MRMC are proposed in Section 6. The simulation study is included in Section 7. Finally in
Section 8 we conclude the paper.

2 Related Work

There has been an increasing interest in enhancing the coverage and connectivity of wireless networks [1], [2], [5]–[9]. Multihop-relaying is usually employed to enhance the transmission performance of primary users (PUs) by selecting secondary users (SUs) as relays in Cognitive Radio Networks (CRNs) [1], [2]. Wang et al. [5] investigate a connectivity-enhancing mechanism for large-scale wireless sensor and ad hoc networks by introducing k-hop clustering. Syue et al. [7] investigate radio coverage extension in physical layer, and focus on finding the impact of physical-layer cooperation on the network-layer routing design. Guizani et al. [8] leverage a cross-layer approach to encourage potential relaying nodes to provide services for others. These existing mechanisms not only extend the wireless coverage area, but also aim to enhance the QoS by increasing the average throughput and decreasing the delay. However, the cost of relays caused by providing services is not perfectly considered in their solutions.

To better utilize the forwarding capability of each node in RUN, resource (e.g., bandwidth, wireless channel) allocation and relay-client association are two key issues that need to be investigated. Several works have been proposed to maximize the revenue of ISP and address the congestion control issue in infrastructure networks. Non-competitive pricing has been extensively employed as a tool to handle resource allocation in the literature ([10]–[12]). Hande et al. [10], as a representative contribution, present an ISP revenue maximization and congestion management scheme for the appropriate choice of the access price for broadband services offered by a monopoly ISP. However, the existing pricing-based solutions mainly focus on maximizing service providers’ revenue in infrastructure networks without considering end users’ profit and somehow fail to capture the full characteristics of RUN (e.g., role exchange). In this paper, we try to explore an applicable solution to guarantee the social welfare of RUN.

On the other hand, relay-client association is similar to AP association that has been widely studied in recent ten years ([13]–[17]). Reference [13], as one of the representative literature, propose a distributed access point selection architecture considering both throughput and fairness. Dawei Gong et al. [14] explore AP association for 802.11n with heterogeneous clients (802.11a/b/g/n) and consider the problem as a MAC layer utility maximization problem. The frame aggregation feature of 802.11n is considered in their solution to avoid poor client throughput and overload APs. Different from them, Nicholson et al. [16] propose a measurement-based solution named Virgil. It performs fast connection and tests on all the available APs to choose the best one (maximum bandwidth, best signal strength, etc.) when service unavailability occurs. In general, the QoS metrics employed in these solutions usually lack of considerations for various user demands. Serving urgent users in priority may gain more social welfare from the entire network perspective.

Resource allocation and relay-client association for cooperative multi-hop access in RUN has not been widely investigated yet, which is the main focus of our paper.

3 Models and Definitions

In our RUN mechanism, the service that the client obtains is the most important factor related to QoC maximization. We employ access bandwidth of a client as a metric to measure the service it receives. Note that in our analysis, we focus on “one unit time” whenever an “amount” is involved for simplicity. For instance, the utility function of a client defined in Definition 2 refers to the satisfaction degree corresponding to the amount of access bandwidth it receives “per unit time”.

Definition 1 (Access bandwidth of a client): The access bandwidth of a client is defined to be the amount of data transmitted by the client per unit time.

The relationship between the access bandwidth and the user’s satisfaction should not be a simple linear function. Here we introduce a utility function to describe such a relationship.

Definition 2 (Utility function of a client): The utility function of a client \( c_i \) is defined to be \( f_i(B) \), where \( B \) is the client’s access bandwidth obtained from its relay and \( f_i(B) \) is the satisfaction degree corresponding to the amount of access bandwidth \( B \). It reflects the urgency degree of the client’s service demand.

Assumption 1: The utility function of a client is concave.

Assumption 1 can be justified as follows. The derivative \( f_i'(B) \) of the function \( f_i(B) \) is equivalent to the marginal utility (MU) of the access bandwidth. According to the Law of Diminishing Marginal Utility [18], MU decreases with the increase of the access bandwidth because a client’s desire of getting more access bandwidth does not increase with each additional unit of bandwidth acquired. Therefore \( f_i'(B) \) is a decreasing function. Thus \( f_i(B) \) should be concave.

Definition 3 (Serving bandwidth of a relay): The serving bandwidth of a relay is defined to be the total amount of bandwidth in bits per second that the relay utilizes as its forwarding service.

A relay’s serving bandwidth is the summation of access bandwidth of all its clients. Clients in RUN can achieve more utility by increasing the relay’s serving bandwidth. However, the forwarding service may have negative impact on the relay’s own transmission. Meanwhile, the energy consumption and CPU utilization may become heavier. Therefore, a relay pays a cost when providing forwarding service, which is formulated by a cost function in our model.

Definition 4 (Cost function of a relay): Denote the cost function of a relay \( r_j \) by \( g_j(B) \), where \( B \) is the relay’s serving bandwidth and \( g_j(B) \) is the corresponding cost.
Assumption 2: The cost function of a relay is convex. 

Assumption 2 can be justified as follows. The derivative \( g_i'(B) \) of the function \( g_i(B) \) is equivalent to the marginal cost (MC) of the serving bandwidth. MC is increasing with the increase of the relay’s serving bandwidth. The reason is that when the occupancy rates of CPU and bandwidth become higher, the harm to the performance of the relay caused by forwarding for others becomes heavier. Therefore \( g_i'(B) \) is an increasing function. Thus \( g_i(B) \) is assumed to be convex.

Definition 5 (QoC of RUN): The QoC (Quality of Cooperation) \( Q \) of a RUN is the metric of the cooperation efficiency among users in RUN and is calculated by the summation of all clients’ utility minus all relays’ cost.

An example of utility function \( f_i(B) \), cost function \( g_i(B) \) and QoC function \( Q_{rc}(B) \) in a single-relay single-client scenario is illustrated in Fig. 1.

![Fig. 1. An example of utility, cost and QoC functions.](image)

**4 SINGLE-RELAY MULTI-Clients**

We first consider a relatively simple scenario in which there is only one relay \( r_j \) serving \( N \) clients in RUN. The \( N \) clients are denoted by \( C = \{c_1, c_2, ..., c_N\} \). Fig. 1 illustrates a simpler case in which \( N = 1 \). It can be observed that the QoC starts to decline when the serving bandwidth exceeds a threshold. Therefore, the relay should limit each client’s maximum access bandwidth by a cutoff bandwidth.

**Definition 6 (Client cutoff bandwidth):** Define the client cutoff bandwidth of a client \( c_i \), denoted by \( B_{ic} \), to be the maximum access bandwidth it is allowed to use, which is limited by its relay.

**Definition 7 (Relay cutoff bandwidth \( B_{rc} \)):** Define the relay cutoff bandwidth of a relay \( r_j \), denoted by \( B_{rc} \), to be the summation of its client cutoff bandwidth.

A set of client cutoff bandwidth, denoted by \( \{B_{ic}\} = \{B_{1c}, B_{2c}, ..., B_{Nc}\} \), should be determined to maximize the total QoC of RUN. However, the QoC cannot be solely determined by the serving bandwidth. It also relates to the access bandwidth of each client that is determined by its bandwidth demand.

**Definition 8 (Bandwidth demand of a client):** Define the bandwidth demand of a client to be the bandwidth required for its current data transmission.

Here we define the critical marginal utility (MU) of a client \( c_i \) to be \( f_i'(B_{ic}) \). Similarly, critical MC of a relay \( r_j \) is defined to be \( g_j'(B_{rc}) \). In the following we discuss how to evaluate the cutoff bandwidth of each client under different bandwidth demand models.

**4.1 Free Bandwidth Demand Model**

We first consider an ideal bandwidth demand model, in which the bandwidth demand of a client has no constraint. Therefore, the relay can evaluate an optimal client cutoff bandwidth allocation to maximize the RUN’s QoC without considering the bandwidth demands of its clients. The total QoC \( Q \) of the RUN can be calculated as follows:

\[
Q = \sum_{i=1}^{N} f_i(B_{ic}) - g(B_{rc}), \tag{1}
\]

where \( B_{rc} = \sum_{i=1}^{N} B_{ic} \), which is the serving bandwidth of the relay. Then the optimal client cutoff bandwidth allocation \( \{B_{ic}\} = \{B_{1c}, B_{2c}, ..., B_{Nc}\} \) to maximize the RUN’s QoC should be the solution to the following problem:

\[
\max_{\{B_{ic}\}} Q \quad s.t. \quad B_{ic} \geq 0 \tag{2}
\]

We have the following theorem:

**Theorem 1:** Under the free bandwidth demand model and the optimal client cutoff bandwidth allocation, the critical MU of each client equals the critical MC of the relay.

It can be formulated by:

\[
\frac{df_i(B_{ic})}{dB_{ic}} = \frac{dg(B_{rc})}{dB_{rc}}, 1 \leq i \leq N \quad s.t. \quad B_{ic} \geq 0 \tag{3}
\]

The proof can be found in the supplementary file 1. To make the theorem more comprehensive, we give an easy-to-understand example in section 4.1 of the supplementary file. In the following part, we utilize this theorem to compute an optimal bandwidth allocation under free bandwidth demand model.

**4.2 Dynamic Bandwidth Demand Model**

We try to consider a more practical bandwidth demand model in this section. In realistic network environments, the bandwidth demand of a client may vary from time to time. For a client \( c_i \), assume that its bandwidth demand is a continuous random variable which follows a certain distribution with a probability density function \( q_i(B) \) (can be derived accordingly to the historical bandwidth demand). The client informs the relay of its \( q_i(B) \), including the function type and the value of each coefficient (the message can be appended to the association request). The relay replies with the client cutoff bandwidth \( B_{ic} \). If the client’s bandwidth demand exceeds \( B_{ic} \) during its data transmission (the probability equals \( \int_{B_{ic}}^{\infty} q_i(B)dB \)), its access bandwidth will be restricted to the client cutoff bandwidth to guarantee social welfare.

---

1. For bandwidth notations such as \( B_i, B_{rc}, B_{min} \) and \( B_{max} \), index is the superscript, i.e., \( B_i \). But for function notations such as \( f, g, h, q \), index is the subscript, i.e., \( f_i \).

2. For all the theorems in the paper, the detailed proofs can be found in our supplementary file.
Under this bandwidth demand model, we evaluate the total QoC from the perspective of mathematical expectations. The expectation of the utility of a client \( c_i \) is 
\[
E_{utility} = \sum_{i=1}^{N} \left( \int_{0}^{B_{c_i}} f_i(B)q_i(B)dB + \int_{B_{c_i}}^{\infty} f_i(B)q_i(B)dB \right) \tag{4}
\]
Thus the expectation of the total utility of \( N \) clients can be expressed by (4).
\[
E_{utility} = \sum_{i=1}^{N} \left( \int_{0}^{B_{c_i}} f_i(B)q_i(B)dB + \int_{B_{c_i}}^{\infty} f_i(B)q_i(B)dB \right)
\]

Meanwhile (5) gives the expectation of the relay’s serving bandwidth.
\[
E_{bs} = \sum_{i=1}^{N} \left( \int_{0}^{B_{c_i}} B q_i(B)dB + \int_{B_{c_i}}^{\infty} B q_i(B)dB \right) \tag{5}
\]

Thus the expectation of the relay’s total cost can be evaluated by (6).
\[
E_{cost} = g(E_{bs}) \tag{6}
\]
That is to say, the expectation of total QoC is
\[
E_{Q} = E_{utility} - E_{cost} \tag{7}
\]

Then an optimal client cutoff bandwidth allocation that can maximize the expectation of the total QoC under the dynamic bandwidth demand model should be the solution to the following problem.
\[
\max_{\{B_{c_i}\}} E_{Q} \quad\text{s.t.} \quad B_{c_i} \geq 0 \tag{8}
\]

The definition of the relay’s critical MC should also be extended from the perspective of mathematical expectations, i.e., \( g(E_{bs}) \) instead of \( g(B_{cr}) \). We have:

**Theorem 2:** Under the dynamic bandwidth demand model and the optimal client cutoff bandwidth allocation, the critical MU of every client equals the critical MU of the relay.

Theorem 2 can be formulated by:
\[
\frac{df_i(B_{c_i})}{dB_{c_i}} = \frac{dg(E_{bs})}{dE_{bs}}, \quad 1 \leq i \leq N \quad\text{s.t.} \quad B_{c_i} \geq 0 \tag{9}
\]

It can be utilized to compute an optimal cutoff bandwidth allocation under dynamic bandwidth demand model from the mathematical expectation perspective.

### 4.3 Capacity Limitation and Minimum Bandwidth Demand Model

The above investigation implies an assumption that the capacity of a relay is infinite and a client can accept an infinitesimally small bandwidth. In reality, a relay’s capacity is always upper-bounded and a client’s bandwidth demand is always lower-bounded. Therefore a relay can not afford to serve an excessive number of clients and a client can not accept a relay that could not supply a minimum bandwidth. Denote by \( B_{max} \) the maximum serving bandwidth of the relay and by \( B_{min} \) the minimum bandwidth demand of client \( c_i \). These two new restrictions make our model more practical. The problem of optimal client cutoff bandwidth allocation can be formulated by:

\[
\max_{\{B_{c_i}\}} Q \quad\text{s.t.} \quad B_{c_i} \geq B_{min}^i \quad\text{and} \quad B_{cr} \leq B_{max}^r. \tag{10}
\]

We have the following theorem,

**Theorem 3:** Under the single-relay multi-client scenario with the capacity limitation and minimum bandwidth demand model, the bandwidth allocation problem defined by (10) is NP-hard.

We prove the theorem by a reduction from the well-known subset sum problem [19] to the above problem. The proof can be found in the supplementary file.

We design a heuristic algorithm named SRMC-ES (Single-Relay Multi-Client based on Equation Solving) to compute a feasible cutoff bandwidth allocation.

**Algorithm 1: SRMC-ES**

**Input:** \( B_{max}, C = \{c_1, c_2, \ldots, c_N\}, \{B_{min}^1, B_{min}^2, \ldots, B_{min}^N\}, r, \{f_1, f_2, \ldots, f_N\}, g \)

**Output:** \( \{B_{c_i}\} \)

1. \( B_{cr} \leftarrow \infty \)
2. while \( B_{cr} > B_{max} \) do
   3. \( \{B_{c_i}\} \leftarrow \text{Band Alloc}(r, C) \)
   4. // Optimal allocation
   5. \( \{B_{c_i}\} \leftarrow \text{Band Alloc}(r, C) \)
   6. // Ensure minimum bandwidth demand of every client
   7. for \( i = 1; i \leq N; i++ \) do
      8. if \( B_{c_i} < B_{min}^i \) then
         9. \( B_{c_i} \leftarrow B_{min}^i \)
      end
   end
   10. // Select the client \( c_{\text{index}} \) with lowest QoC contribution
   11. for each \( c_i \) in \( C \) do
      12. \( \delta_i \leftarrow \text{Compute Delta}(c_i) \)
   end
   13. index \( \leftarrow \arg \min \{\delta_i, \forall c_i \in C\} \)
   14. // Stop serving \( c_{\text{index}} \)
   15. \( B_{cr} \leftarrow (B_{cr} - B_{\text{index}}) \)
   16. \( B_{\text{index}} \leftarrow 0 \)
   17. if \( B_{cr} \leq B_{max} \) then
      18. break
   end
   19. \( C \leftarrow C - \{c_{\text{index}}\} \)
end
1. return \( \{B_{c_i}\} \)

The design motivation of Alg. 1 is stated as follows. We first get the optimal bandwidth allocation \( \{B_{c_i}\} \) without the capacity and demand limitation by solving equations obtained from (3) or (9) (\( \text{Band Alloc}(r, C) \)). If the relay can not afford to serve all the clients while keeping the RUN’s QoC as high as possible, the clients with relatively low contributions to the RUN’s QoC will not be served. Here we introduce a metric \( \delta_i \) to quantify \( c_i \)’s contribution to the RUN’s QoC.
\[
\delta_i = f_i(B_{c_i}) - (g(B_{cr}) - g(B_{cr} - B_{c_i})), \tag{11}
\]
where \( g(B_{cr} - B_{c_i}) \) is part of the relay’s cost caused by the clients other than \( c_i \). Thus \( \delta_i \) stands for \( c_i \)’s portion in the RUN’s QoC.

We denote the time complexity of \( \text{Band Alloc}(N) \), which depends on the approach adopted to solving (3) or (9). \( \text{Band Alloc}(\cdot) \) is called at most \( N \) times.
in the main loop of SRMC-ES. Thus the time complexity of SRMC-ES is \( T(\text{SRMC-ES}) = O(N \cdot T_B(N)) \).

### 4.4 An Approximation Solution for Bandwidth Bounded Model

The major contribution to the running time of Alg. 1 is the time used to solve (3) or (9), i.e. \( T_B(N) \). It highly relies on the exact forms of utility and cost functions, as well as the algorithm employed to solve the differential equations. To reduce the possible uncertainty and instability, we further propose a dynamic-programming-based approximation algorithm that does not need to solve (3) or (9), while achieving an approximate solution at an acceptable approximation degree.

**Algorithm 2: SRMC-DP**

**Input:** \( B_{\text{max}}, \mathcal{C} = \{c_1, c_2, \ldots, c_N\}, \{B_{\text{min}}^1, B_{\text{min}}^2, \ldots, B_{\text{min}}^N\}, r, \{f_j, j \in \mathcal{N}\}, g \)

**Output:** \( \{B_i^j\}, Q \)

```plaintext
1: \( B_{\text{min}} \leftarrow \min_i B_{\text{min}}^i \)
2: \( Q \leftarrow \infty \)
3: \{\{B_i^j\}\} \leftarrow 0
4: for \( B_r = B_{\text{min}}, B_r \leq B_{\text{max}}; B_r = B_r + \Delta B \) do
   // Initialization
   5: \( dp(0, j) = 0, \forall j \in \mathcal{N} \)
   6: for \( i = 1; i \leq N; i++ \) do
      7: for \( j = B_{\text{min}}^i; j \leq B_r; j = j + \Delta B \) do
         8: \( dp(i, j) = 0 \)
      end
   end // Dynamic Programming
   // \( i \) \( 1 \) \( \Delta B \)
   10: for \( j = B_{\text{min}}^i; j \leq B_r; j = j + \Delta B \) do
      11: for \( k = B_{\text{min}}^i; k \leq \min \left\{ B_r, j \right\} = k + \Delta B \) do
         12: \( dp(i, j) = \max_k \{ dp(i, j - k) + f_j(k) \} \)
      end
   end // \( j \) \( 1 \) \( \Delta B \)
   14: \( U_{\text{max}} = dp(N, B_r) \) according to the recorded \( k_i \)
   15: if \( Q < \min \{ U_{\text{max}} - g(\sum_{j=1}^{N} B_i^j), Q \} \) then
      16: \( Q \leftarrow U_{\text{max}} - g(\sum_{j=1}^{N} B_i^j) \) according to \( \{B_i^j\} \)
   end
   19: Update \( \{B_i^j\} \) according to the recorded \( k_i \)
21: return \( \{B_i^j\} \) and \( Q \)
```

As shown in Fig. 2(a), we leverage piecewise linear functions to approximate the utility of clients and the cost of relay. Since the minimum bandwidth demand and the capacity of the relay are the lower bound and upper bound of client \( i \)'s cutoff bandwidth, respectively, we only discretize the utility and cost function of each client \( i \) from \( B_{\text{min}}^i \) to \( B_{\text{max}}^i \). With piecewise granularity being \( \Delta B \), the cutoff bandwidth of client \( i \) can only be chosen from the set \( \{0, B_{\text{min}}^i, B_{\text{min}}^i + \Delta B, \ldots, B_{\text{max}}^i\} \).

The basic idea of the heuristic algorithm is depicted as follows. We first maximize the total utility of all clients \( \sum_{i=1}^{N} f_i(B) \) with the bandwidth capacity of the relay being \( B_{\text{max}} \). Unlike the situation in the NP-hard proof above, the QoC of the RUN will not always be maximized even if the capacity of the relay is used up. Providing more serving bandwidth may cause negative QoC income with the influence of the cost of the relay. Therefore, we repeat the process of maximizing the total utility under different relay capacities \( \{B_{\text{min}}, B_{\text{min}} + \Delta B, \ldots, B_{\text{max}}\} \), where \( B_{\text{min}} \) is the minimum bandwidth demand among the \( N \) clients. Then we choose the maximal QoC from the results obtained under different capacities.

First, we propose a dynamic programming based algorithm to address the utility maximization problem. As shown in Fig. 2(b), under specific bandwidth capacity, the problem can be regarded as a \( 0-1 \) knapsack problem on a rooted tree \( T \) such that if a node is selected into a knapsack, then all nodes on the path from the selected node to the root node must also be selected into the knapsack. Here the vertices in the tree (the items of knapsack problem) represent the different cutoff bandwidth, the price of each item is the utility, the \( N \) branches represent \( N \) clients, and the weight capacity of the knapsack is the bandwidth capacity of the relay. The Tree-Knapsack Problem (TKP) has been proved to be weakly NP-complete [20]. In this paper, we employ dynamic programming to address the problem in pseudo-polynomial time.

In the dynamic programming process of Alg. 2, we assume the total utility in state \((i, j)\) to be \( dp(i, j) \), where \( i \) represents client 1 to client \( i \), and \( j \) represents the remaining bandwidth capacity of the relay. The parameter \( k \) in lines 13-16 is the bandwidth allocated to client \( i \). Note that client \( i \) may not be served by the relay, in which case the sub-state is \( dp(i - 1, j) \).

The time complexity of the dynamic programming process (lines 11-18) is \( O(N B_{\text{max}}^2) \). The process is invoked under different bandwidth capacities, hence the running time of Alg. 2 is \( O(N B_{\text{max}}^3) \), which is pseudo-polynomial.

### 5 Multi-Relay Multi-Client

In this section we study the scenario of multiple relays and multiple clients in RUN. The following two issues need to be investigated: how to associate clients with relays and how to allocate client cutoff bandwidth.

**Definition 9 (Cluster):** A cluster \( i \) is defined to be the node set consisting of relay \( i \) and all the clients it serves. The serving relationship may vary from time to time.

Assume that there are \( K \) relays denoted by \( R = \{r_1, r_2, \ldots, r_K\} \). Define by \( X = (x_{ij}) \) a binary association
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPDS.2014.2308201, IEEE Transactions on Parallel and Distributed Systems

matrix with \( x_{ij} = 1 \) if and only if \( r_i \) serves \( c_j \). Let \( B = (b_{ij}) \) be a client cutoff bandwidth matrix with \( b_{ij} \) being the cutoff bandwidth assigned to \( c_j \) by \( r_i \). If the free bandwidth demand model is utilized, the QoC \( Q_i \) of each cluster \( i \) can be evaluated by

\[
Q_i = \sum_{j=1}^{N} x_{ij}f_j(b_{ij}) - g_i(\sum_{j=1}^{N} x_{ij}b_{ij})
\]

Then the client-relay association and the client cutoff bandwidth allocation issue can be formulated as follows.

\[
\max_{X,B} \sum_{i=1}^{K} Q_i
\]

s.t.

\[
\sum_{i=1}^{N} x_{ij} = 1, \quad 1 \leq j \leq N
\]

\[
\sum_{j=1}^{N} x_{ij}b_{ij} \leq B_{i,\text{max}}', \quad 1 \leq i \leq K
\]

\[
\sum_{j=1}^{N} x_{ij}b_{ij} \geq B_{i,\text{min}}', \quad 1 \leq i \leq K
\]

\[
x_{ij} \in \{0, 1\}, \quad 1 \leq i \leq K \text{ and } 1 \leq j \leq N
\]

In (13), the association matrix \( X \) and the client cutoff bandwidth matrix \( B \) are the variables to be computed. The objective function is to maximize the summation of all clusters’ QoC. The first constraint indicates that every client can get forwarding service from one relay; the second constraint indicates that the maximum serving bandwidth of the relay \( r_i \) can not exceed its capacity limitation \( B_{i,\text{max}}' \) while the third constraint indicates that the received bandwidth of a client \( c_j \) should not be smaller than its minimum bandwidth demand \( B_{i,\text{min}}' \). We have the following theorem.

**Theorem 4:** The client-relay association and the client cutoff bandwidth allocation issue under MRMC is NP-hard.

The proof can be found in the supplementary file.

We propose a greedy algorithm Alg. 3 to compute a feasible solution for (13). We first examine all possible relay-client pairs to find out the one with the highest contribution to the network QoC. Then the client is associated with that relay. Here we use \( \delta_j \) defined in (11) as a metric to quantify the QoC contribution of the client. This process is repeated until all clients are associated with relays or all relays are at full capacity.

The while-loop of MRMC is executed at most \( N \) times, assuming \( N > K \). SRMC-ES is called \( N \cdot K \) times during the first iteration and at most \( N \) times in each of the other \( N - 1 \) iterations. Thus the time complexity of MRMC is \( T(MRMC) = O(N(N+1) + NK) \cdot O(N)T_B(N) = O(N^2(N + K)) \).

3. Note that the algorithm employed to compute \( \delta_j \) and \( \{B_{ij}'\} \) can also be replaced by SRMC-DP (line 7).

**Algorithm 3:** MRMC

```
Input: \( C = \{c_1, c_2, ..., c_N\}, \{r_1, r_2, ..., r_K\}, \{f_1, f_2, ..., f_N\}, (g_1, g_2, ..., g_K), (B_{i,\text{max}}', B_{i,\text{min}}'), (B_{i,\text{max}}^1, B_{i,\text{min}}^1, ..., B_{i,\text{min}}^N) \)

Output: \( X = (x_{ij}), B = (b_{ij}) \)
// Define \( C_r \) as the set of clients currently served by \( r_i \)
for each \( c_j \) in \( C \) do
   \( \{\delta_j, B_{c_j, r_i}^\ell, \nu(r_i)\} \leftarrow \text{SRMC-ES}(r_i, C_r, \nu(r_i), c_j) \)
   \( \delta_j \leftarrow \arg \max \{\delta_j, \nu(r_i)\} \)
   \( C_r \leftarrow C_r \cup \{c_j\} \)
end
// Update \( C_r \) and the bandwidth for each \( \{c, r\} \) pair
foreach \( r_i \) in \( R \) do
   if \( \text{index}_r = 0 \) or \( \text{index}_r = \text{index}_c \) then
      \( \{\delta_j, B_{c_j, r_i}^\ell, \nu(r_i)\} \leftarrow \text{SRMC-ES}(r_i, C_r, \nu(r_i), c_j) \)
   for each \( c_j \) in \( C_r \) do
      \( \text{index}_r \leftarrow \text{index}_r + 1 \)
      \( B_{c_j, r_i}^\ell \leftarrow B_{c_j, r_i}^\ell + \delta_j \)
   end
end
// A relay at full capacity should not serve any more clients
if \( B_{c_j, r_i}^\ell \geq B_{c_j, r_i}^\text{max} \) then
   \( R_i \leftarrow R_i - c_j \)
end
return \( (X, B) \)
```

**6 DISTRIBUTED DYNAMIC MECHANISM OF MRMC**

The MRMC algorithm presented in Section 5 is a centralized one, which has two limitations. First, the computation process should be executed on a certain centralized control node. The overhead of gathering information from the whole network might be quite high and sometimes impractical. Second, it is difficult to handle the dynamic arrivals and departures of the members in RUN, which requires the execution of the algorithm to correspondingly update the association and bandwidth allocation of the whole network whenever there is a RUN member change.

Therefore, in this section we propose a distributed dynamic mechanism D^2MRMC for the MRMC scenario. D^2MRMC consists of a protocol that defines the signaling process between clients and relays, and several modified algorithms to determine relay-client association and bandwidth allocation.

As shown in Fig. 3, we extend the standard 802.11 AP-STA association protocol [21] to handle client-relay association. The standard beacon frame defined in 802.11 is extended by appending an additional **Cooperation Pa-**
rameter Set after the IBSS Parameter Set. The Cooperation Parameter Set contains the cost function and the capacity limitation of the relay, and the utility functions, the minimum bandwidth demands, etc.. When a client moves out of the coverage range of any AP and seeks for forwarding service, it can receive a list of available relays by listening to their beacon frames. The client determines which relay is the most appropriate one, sends an Association Request frame and gets the Association Response frame to/from the relay to complete the association procedure.

Fig. 3. The signaling process of client-relay association

We divide our MRMC algorithm into three components to meet the requirement of the distributed scenario. Alg. 4 is executed on each client to determine its expected relay and the client cutoff bandwidth allocated by that relay when it needs the forwarding service. Alg. 5 and Alg. 6 are executed on each relay, with the former one handling the arrival of clients, and the latter the departure. If a newly arriving client has a direct access to an AP, it can work as a relay by broadcasting the beacon frames to attract clients; when it leaves the network, the associated clients need to call Alg. 4 to search for a new relay.

Algorithm 4: D²MRMC: Client Access

Input: $c_j, R = \{r^1, r^2, ..., r^K\}$, Cooperation Parameter Set for each $r^i$  
// output is the index of the relay that client $c_j$ chooses to associate with  
Output: index

1. if $c_j$ needs forwarding service but has no relay then
2. for each $r_i \in R$ do
3. \{\{\delta_i, B^{C_{r_i}} \cup \{c_j\}\}\} \leftarrow \text{SRMC-ES}(r_i, C_{r_i} \cup \{c_j\}, c_j)
4. end
5. index \leftarrow \text{arg max} \delta_i, \forall r_i \in \{r_i\}
6. $c_j$ connects to $r_{\text{index}}$
7. end

Alg. 4 presents the access procedure executed on each client $c_j$. For each relay $r_i$, the client $c_j$ computes its QoC contribution and the corresponding bandwidth allocation based on the Cooperation Parameter Set (line 3). The client selects the relay which contributes the most to the network QoC according to $\delta_{ij}$ defined in (11).

Alg. 5 details the procedure of handling the arrival of a client at each relay $r_i$. If a client decides to connect to $r_i$, $r_i$ gets the bandwidth allocation vector $B_r^{C_{r_i}}$ from the client and updates the client cutoff bandwidth of each client it currently serves by sending the Association Response frame. If the capacity of $r_i$ is full, it would reject the association request of any newcomers, and stop its broadcast of the beacon frames.

Capacity release (lines 7-14) is an option to increase the QoC of the network if a newly arriving client can bring more QoC than any existing one(s). To alleviate QoC reduction caused by the clients that contribute less and come earlier, we need to release the capacity of the relay by kicking out these clients (lines 11-13). The clients could call Alg. 4 again to select a new relay.

Algorithm 6: D²MRMC: Client Departure

Input: $r_i, C_{r_i}, B_r^{C_{r_i}}$
Output: The updated $C_{r_i}$ and $B_r^{C_{r_i}}$

1. if $B_r^{C_{r_i}}$ was served by $r_i$ and it leaves the coverage of $r_i$ then
2. \{$C_{r_i} \leftarrow (C_{r_i} - \{c_j\})$\}
3. $B_r^{C_{r_i}} \leftarrow \text{SRMC-ES}(r_i, C_{r_i})$
4. for each client $c_k \in C_{r_i}$ do
5. update $B_r^{C_{r_i}}$ according to $B_r^{C_{r_i}}$
6. end
7. end

When a client moves out of the coverage area of its relay, the relay should detect this departure event and call Alg. 6 to re-calculate the client cutoff bandwidth for other serving clients. Then it announces the updated bandwidth allocation to its clients through the Association Response frame. This client-departure handling procedure makes D²MRMC more flexible especially when users frequently move in and out.

7 Evaluations

In this section we evaluate our RUN framework by simulation study. We first consider the case of a single relay and multiple clients to evaluate the QoC income under the optimal allocation, SRMC-ES, and SRMC-DP.
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPDS.2014.2308201, IEEE Transactions on Parallel and Distributed Systems

---

7.1 SRMC Performance

We consider the case of single-relay multi-client first. The results for 1 to 7 clients are reported in Fig. 4(a), 4(b) and 4(c). The legend “Optimal” in Fig. 4(a) and Fig. 4(b) stands for the optimal cutoff bandwidth allocation obtained from Theorems 1 and 2. “Average” and “Fixed” in Fig. 4(c) are listed for comparisons, with “Average” indicating that the relay’s serving bandwidth is equally allocated to all clients and “Fixed” implying that the cutoff bandwidth of each client is fixed to 2Mbps.

The free bandwidth demand model is employed in Fig. 4(a). The utility functions for the seven clients (from client #1 to #7) are defined by $2\sqrt{B}, 3.5\sqrt{B}, 5\sqrt{B}, ..., 11\sqrt{B}$ (step 1.5) respectively, while the cost function of the relay is $0.2B^2$. It is clear that our allocation scheme leads to a higher network QoC than the other two schemes.

The result of the dynamic bandwidth demand model is shown in Fig. 4(b). Each data point is an average over 50 runs to reduce randomness. The utility functions are the same as in Fig. 4(a), and the cost function is $0.4B^2$. The bandwidth demand of a client follows a uniform distribution on [0.5, 2.5] Mbps. It can be observed that our scheme still achieves the maximum QoC, while “Fixed” performs bad compared with the other two schemes.

The result of the bandwidth bounded model is shown in Fig. 4(c), in which SRMC-ES and SRMC-DP are evaluated. Each data point is an average over 50 runs to reduce randomness. The bandwidth capacity of the relay is fixed to 7Mbps, while the minimum bandwidth demand of each client follows a uniform distribution on [0.5, 2.5] Mbps. The utility functions for the seven clients and the cost function for the relay are the same as in Fig. 4(b). Note that our SRMC-DP achieves a higher QoC compared to the other three schemes. However, since the running time of SRMC-DP highly relies on the value of $B_{max}$ and the bandwidth granularity ($\Delta B$), it may take an unacceptable amount of time on bandwidth allocation compared to SRMC-ES under specific circumstances.

During the simulation of SRMC-ES, some of the clients may be kicked out from obtaining relay’s service because they contribute relatively little QoC to the network. We also calculate the average proportions of the kicked clients in Fig. 4(d). With the increase of the total number of clients, the proportion of the relay’s serving clients (marked as “Total” in the figure) becomes smaller. This is reasonable since relay’s capacity is upper-bounded, and the relay has to stop serving these clients to achieve more utility from the clients with larger contributions. The statistics of the kick rate of client #1, 2, 3, 4 also proves it. With the increase of the total number of clients, the clients with less contributions (e.g. $2\sqrt{B}$ for Client #1) will be kicked out with a higher priority.

7.2 MRMC Performance

Fig. 5(a) and 5(b) depict the simulation results of MRMC in comparison with “Random-Average” and “Random-Fixed” schemes. "Random" implies that the clients are randomly associated with relays. “Average” implies that the relays allocate its bandwidth capacity to all clients equally, and “Fixed” implies that the relays allocate fixed bandwidth to each client until their bandwidth capacity is used up. The utility functions of the clients are defined as $\sqrt{B}, 3\sqrt{B}, 5\sqrt{B}, ..., 23\sqrt{B}$ (step 2) while the cost functions of the relays are $0.1B^2, 0.15B^2, 0.2B^2, ..., 0.35B^2$ (step 0.05). The bandwidth capacities of the relays follow a uniform distribution on [6,14] Mbps, while the minimum bandwidth demands of the clients follow a uniform distribution on [0.5,2.5] Mbps. Fig. 5(a) reports the total QoC under a various number of clients while the number of the relays is 4. Fig. 5(b) shows the result under a various number of relays while the number of the clients is fixed to 8. The result of MRMC is an average over 50 runs while those of “Random-Average” and “Random-Fixed” are over 1000 runs to reduce randomness. It is obvious that MRMC possesses an apparent advantage over the other two schemes. “Random-Average” has very poor performance when the number of clients is small or the number of relays is large. This is because each relay serves few clients in these two circumstances, and the bandwidth allocated to each client may be quite high, which may cause pretty high relay cost.

Next, a small public area WLAN is simulated in which 30 users locate in the area and form a RUN (note that here node arrival and departure are not considered).
A user in RUN both transmits its own data and provides forwarding service for others. We compare the average amount of data transmitted by relays and clients (including relays’ own data) and the average amount of service provided by relays during the 30-day period in Fig. 5(d). The difference between the average amount of data transmitted and the average amount of service provided becomes larger when the proportion of the relays is higher, which indicates that more users can connect to APs directly under this scenario.

### 7.3 $D^2$MRMC Performance

To validate $D^2$MRMC, we simulate a small public area WLAN in which 30 users randomly visit a circular area everyday, making up RUN. There only exists one AP in the center of the area. On each day, users arrive at and depart from the area with a preset probability $p$. Each user has a random distance from the AP, which determines whether the user has a direct AP access or has to obtain forwarding service from relays. The ratio of the AP coverage radius to the radius of the whole circular area, which is designated as coverage ratio, varies from 0.2 to 0.7. The communication overhead (latency) between clients and relays is a fixed value. The relays can obtain 7Mbps bandwidth from the AP. A client can connect to a relay only when their distance, which is determined by their random positions, is smaller than a certain value. The mobility pattern of each node follows a random distribution, the $x,y$ coordinates of each node are generated by a $\text{rand}(\cdot)$ function. Other parameter settings are the same as those presented in Section 7.2 for validating MRMC. The simulation results are reported in Fig. 6.

We calculate the average access bandwidth of all users over the 30-day period and report the results in Fig. 6(a). It can be observed that the average bandwidth increases by 25%-50% with the RUN mechanism using $D^2$MRMC compared with “No RUN”. When the coverage ratio is lower, the increase of the average bandwidth under the RUN mechanism is more obvious. The “Random-Average” achieves larger average bandwidth than $D^2$MRMC, but the cost of relays are also very large just as we have depicted in Fig. 5(c).

Fig. 6(b) illustrates the number of users in 4 different user classes per unit time under 6 different coverage ratios. “Potential relay” refers to the users having direct AP access, while “Potential client” refers to the users staying out of the coverage of the AP. Accordingly, “Real relay” indicates the users who are forwarding data for others, while “Real client” stands for the users who are transmitting data through real relays. The number of potential relays per unit time grows and the number of potential clients per unit time decreases with the increase of the coverage ratio. This is because a user has a larger possibility to become a relay rather than a real client if the AP’s coverage is becoming larger. Real clients are less than potential clients, especially when the coverage

---

**Fig. 5. MRMC performance**

On each day, these users are randomly divided into 4-8 groups around different APs (i.e. each group stands for the coverage area of one AP). Users of each group are then randomly divided into relays and clients, and one of them is chosen as the decision node. The proportion of relays in each group varies from 10% to 60%. An operation of 30 days is simulated. The centralized MRMC algorithm is executed on the decision node of each group to compute the association and bandwidth allocation.

The utility functions of the clients are defined as $\sqrt{B, 3\sqrt{B}, ..., 25\sqrt{B}}$ (step 2) while the cost functions of the relays are $0.1B^2, 0.15B^2, 0.2B^2, ..., 0.35B^2$ (step 0.05). The bandwidth capacities of the relays follow a uniform distribution on [6,14] Mbps, while the minimum bandwidth demands of the clients follow a uniform distribution on [0.5,2.5] Mbps. The simulation results are reported in Fig. 5(c) and 5(d).

We calculate the average access bandwidth of all users (shown as bar graph) and the cost of all relays (shown as broken line graph) over the 30-day period in Fig. 5(c). It can be observed that the average bandwidth is increased by 40%-90% with our MRMC algorithm compared with “No RUN”, which demonstrates that the users have more chances to get online by multi-hop access and the network QoC is improved. When the proportion of relays is lower, the increase of the average bandwidth under the RUN mechanism is more obvious. It demonstrates that our solution is more suitable for the public area WLANs with fewer APs and a larger number of users that are far from any AP. Note that the “Random-Average” scheme achieves the maximum average bandwidth, it is reasonable since the bandwidth capacity of each relay is over used by clients in this scheme. From the figure we could see that the cost of relays of “Random-Average” is nearly 3x larger than that of our MRMC algorithm. It proves that our RUN mechanism can achieve nearly the same average bandwidth while considering service providers’ cost simultaneously.

---

**Fig. 6.**

- (a) Number of clients varies
- (b) Number of relays varies
- (c) Average bandwidth
- (d) Average data and service

---

**Tables:**

- MRMC Performance
- QoC of RUN
- Number of clients/Number of relays
- Relay cost
- Average bandwidth
- Average data
- Average service

---

**Figures:**

- MRMC performance
- QoC of RUN
- Number of clients/Number of relays
- Relay cost
- Average bandwidth
- Average data
- Average service

---

**Equations:**

- \( B, 3\sqrt{B}, ..., 25\sqrt{B} \)
- \( 0.1B^2, 0.15B^2, 0.2B^2, ..., 0.35B^2 \)
- \( \sqrt{B, 3\sqrt{B}, ..., 25\sqrt{B}} \)

---

**References:**

- IEEE Transactions on Parallel and Distributed Systems
- DOI 10.1109/TPDS.2014.2308201
- IEEE Transactions on Parallel and Distributed Systems
This article has been accepted for publication in a future issue of this journal, but has not been fully edited. Content may change prior to final publication. Citation information: DOI 10.1109/TPDS.2014.2308201, IEEE Transactions on Parallel and Distributed Systems

8 Conclusion

This paper proposes a cooperative coverage extension mechanism in Relay-Union Networks. By investigating the relationship among bandwidth, cost, and utility, we present a QoC model and formulate the maximization of network's QoC as an optimal cutoff bandwidth allocation problem. We derive the solution to the problem under two different bandwidth demand models. We then extend the problem by taking each relay's capacity limitation and each client's minimum bandwidth demand into account. We design SRMC-ES and SRMC-DP to evaluate a feasible bandwidth allocation for the single-relay multi-client scenario and the MRMC algorithm to compute a relay-client association and bandwidth allocation for the multi-relay multi-client case. Finally, we propose an extended MRMC mechanism termed D²MRMC, which consists of a protocol defining the signaling process, and several algorithms dynamically handling the cooperation problem in a distributed way. Extensive simulation results demonstrate the increase of the network QoC and the per-user bandwidth of our RUN framework.

References

Yong Cui is a professor in Tsinghua University, Council Member in China Communication Standards Association, Co-Chair of IETF IPv6 Transition WG Softwire. Having published more than 100 papers in refereed journals and conferences, he is also the winner of Best Paper Award of ACM ICUIMC 2011 and WASA 2010. Holding more than 40 patents, he is one of the authors in RFC 5747 and RFC 5565 for his proposal on IPv6 transition technologies. His major research interests include mobile wireless Internet and computer network architecture.

Xiao Ma received his bachelor degree in the Department of Computer Science and Technology from Tsinghua University, China in 2011. He is pursuing the master degree in the Department of Computer Science and Technology at Tsinghua University, supervised by Prof. Yong Cui. His research interests include mobile cloud computing, wireless networks and computer network architecture.

Xiuzhen Cheng is an associate professor at the Department of Computer Science, The George Washington University, Washington DC. Her current research interests focus on cognitive radio networks and wireless mobile computing. She has served on the editorial boards of several technical journals and the technical program committees of various professional conferences/workshops. She also has chaired several international conferences. She worked as a program director for the US National Science Foundation (NSF) from April to October in 2006, and from April 2008 to May 2010. She received the NSF CAREER Award in 2004. She is a senior member of IEEE and a member of ACM.

Yihua Guo received his bachelor degree in the Department of Computer Science and Technology from Tsinghua University, China in 2012. He is pursuing the PhD degree at University of Michigan. His research interests mainly include data center networking, wireless networks and mobile systems.

Biao Chen received his BS in Computer Science from Fudan University in China and MS in Mathematics and PhD in Computer Science from Texas A&M University respectively. After graduation, he joined the Department of Computer Science in University of Texas at Dallas as assistant professor. Currently, he is a visiting professor in the Department of Computer and Information Science of University of Macau. His research interests include distributed systems, networking, internet of things, and security. He is a member of Sigma Xi, IEEE, and ACM.