Multicast with cooperative gateways in multi-channel wireless mesh networks

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A B S T R A C T

Wireless mesh networks (WMNs) are emerging as a promising solution for pervasive and cost-effective broadband connections. In this paper, we target high-throughput multicast that combats the interference and bandwidth limitation of wireless channels, which are particularly severe with wireless meshes. We suggest that they can be addressed by introducing multiple cooperative mesh gateways and exploiting the diversity of wireless channels. We present a cross-layer design that jointly selects appropriate channels for each mesh node to use at judiciously tuned power, and computes the optimal multicast flows from multiple cooperative gateways. We show that this design can be iteratively optimized through Lagrange relaxation and primal–dual decomposition. A progressive channel assignment and power level adjustment heuristic is introduced in the MAC/PHY layer, together with a smart link capacity allocation for cooperative gateways in the network layer. Through extensive simulations, we demonstrate the effectiveness of the proposed solution framework and the sub-problem heuristics. In particular, a throughput improvement of up to 100% is observed compared to straightforward approaches of utilizing multiple wireless channels for multicast routing.

1. Introduction

Wireless mesh networks (WMN) are emerging as a promising solution for broadband connectivity, due to its flexibility and cost-effectiveness in bringing a large number of users online, in comparison to competing solutions that depend on a wireline infrastructure [2,5]. In a WMN, Internet gateways, mesh routers and client nodes are organized into a mesh topology. Data flows are routed between the clients and the gateways through wireless links, in a multi-hop fashion. A notable challenge in WMN is to provide support for multicast applications that surged on the Internet during the past decade, such as file dissemination, video conferencing and live media streaming. Such applications usually serve a large number of users, and consume high network bandwidth.

We consider two techniques for addressing the high-throughput requirement of multicast applications in WMNs. The first is to use multi-gateways. A gateway is directly connected to the Internet, and hence serves as the data source for users in a WMN. A single gateway design makes the gateway node a bottleneck, and is prone to congestion during high network activities. Having multiple gateways can dramatically improve the network performance at a reasonable cost. These gateways can collaboratively serve their clients using minimal signalling among the gateways. The second is to exploit the diversity in wireless channels, and provide a multi-channel multicast solution. Wireless interference is a critical limitation on throughput of WMN applications [18]. Utilizing distinct channels at neighbouring nodes for transmission can help

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reduce interference to minimum. For example, the IEEE 802.11b/g protocol defines 13 channels within a 2.4 GHz frequency band [9]. The further apart two channels are, the lower interference exists between them; in particular, channels 1, 6 and 13 are totally orthogonal. However, only considering the orthogonal channels will result in collision and reduced throughput, especially in more dense networks like urban areas. We consider using non-orthogonal channels and show that judiciously assigned, this selection will result in lower interference and enhanced performance.

We first formulate the multi-gateway multi-channel multicast problem in WMNs as a mathematical programming problem, which jointly considers channel assignment and transmission power tuning at the MAC/PHY layer, as well as multicast routing at the network layer. Two important regions that the formulation is based on, the channel capacity region and the routing region, are both convex. Furthermore, the objective function that models the utility of multicast throughput is strictly concave. Therefore, the entire optimization model we obtain is a convex program, if we can freely select the frequency band for a channels. However, with pre-defined channels such as in IEEE 802.11, the optimization model contains discrete variables, which complicates the solution design.

In order to provide an efficient and practical solution to the optimization model, we apply the classic Lagrange relaxation technique [7,31], and derive an iterative primal–dual optimization algorithm that leads to a cross-layer multicast solution. Towards this direction, we first relax the link capacity constraints that couple the channel region and the routing region, and decompose the overall optimization into two smaller sub-problems, one for channel assignment at the MAC/PHY layer, and one for multicast routing at the network layer. Our primal–dual solution framework then iteratively refines the primal solution, with help of the Lagrange dual that signalizes capacity demand at each wireless link. The dual is updated during each iteration based on the latest primal solutions.

To complete the solution defined by the primal–dual framework, we need to precisely define the channel region and the routing region, and design a solution algorithm for each of the channel assignment and routing sub-problems. We formulate the channel assignment problem as a mathematical program, in which channel capacities are computed from their signal-to-noise-and-interference ratio (SINR), and the computation of SINR in turn appropriately takes into account the separation between different wireless channels used at neighbouring mesh nodes. The main challenge in solving this mathematical program lies in the presence of discrete channel assignment variables. We design an efficient heuristic, progressive channel assignment, for overcoming this difficulty. Finally, we discuss both multicast tree based and network coding based solutions for the multicast routing sub-problem. Extensive simulations, with various network sizes, were conducted for evaluating the effectiveness of both the overall primal–dual optimization framework and the sub-problem solutions. Throughput improvement of up to 100% were observed, when the proposed solution is compared to straightforward channel assignment schemes such as orthogonal channel assignment and consecutive channel assignment.

The rest of the paper is organized as follows. We review related research in Section 3. Section 4 presents the optimization problem formulation for multicast in WMN. Section 5 introduces the problem decomposition and the overall primal–dual solution framework. Section 6 presents solutions for the sub-problems. Section 7 is simulation results and Section 8 concludes the paper.

2. Motivating example

In this section, we present an example to illustrate the importance of multicast channel assignment in multi-channel wireless networks. Fig. 1 illustrates a mesh network consisting of 25 nodes within a 150 m × 150 m area. The network contains four gateways providing the connectivity to the same multicast source. Multicast receivers, D1–D7, each wishes to connect to one of the available gateways to access multicast data. We assume using 802.11 g in an indoor environment. The effective communication

![Motivating example: Multi-gateway multicast in a WMN in a crowded area, using orthogonal channels: (a) Hop count-based multicast. (b) Multicast flow and corresponding channel assignment based on our optimization framework.](image)
range of each wireless link in 802.11 g indoor mode is about 35 m, and the interference range is about three times the communication range.

A simple solution to this multi-gateway problem is hop-count based multicast, which builds a multicast forest in the wireless network by connecting each receiver to its closest gateway. This results in connections from D1, D5 and D6 to G1, from D2 to G2, from D3 to G3 and from D7 and D4 to G4. Orthogonal channel assignment on such multicast routes is shown in the Fig. 1a. Numbers on the active wireless links represent the channel assignment on each link.

Orthogonal channel assignment will assign channel 1, 6, 11 with lowest interference detection on each, starting from the source, extending all the way to the destinations. The throughput on the links to the multicast receivers D5–D7 will be rather low, imposing the multicast rate limit on all nodes on the same multicast tree. The low rate is due to the fact that the same channel is re-used in the interference range. If the scanning and scheduling techniques are implemented, the throughput will be as low as half of average 802.11 g throughput.

Given the same flow routing scheme in the wireless network, our proposed channel assignment algorithm, discussed in detail later, will instead produce a solution shown in Fig. 1b. We note that this solution does not insist on using orthogonal channels only, and systematically selects one of the available channels for each link in the flow, for minimizing global interference. As we will discuss later, further coupled with power adjustment module proposed in this work, our channel assignment algorithm can enhance the multicast throughput up to 3/4 of average 802.11 g throughput.

Similar network topology and scenario may exist in crowded, dense urban area wireless networks. The number of wireless users and the interference levels will also be substantially higher in such scenarios due to the fact that usually no node in a WMN is idle. Higher interference levels might also be a result of other working wireless devices in 2.4 GHz range. Our proposed algorithm increases the multicast throughput up to 100% in such scenarios. Later, we will discuss in detail the improvements each part of our algorithm will make on these specific examples.

3. Related work

There have been significant research on wireless mesh networking in recent years [3,6,8,14,17,26,29]. Minimizing WMN installation costs [3], benefits of multi-gateway wireless mesh networks [27], gateway placement [14,8], router placement [30], and relay selection [29] have been discussed in the literature. Channel assignment has consistently been a focus with diverse static and dynamic solutions being proposed [25,16,11,22,23,10,27,1]. Adya et al. [1] proposed a greedy algorithm for channel assignment at each node. A fixed channel assignment for orthogonal channels has also been proposed [10]. Raniwala et al. [27] presented a greedy load-aware channel assignment algorithm for 802.11-based WMN which leads to orthogonal channel assignment were possible and collision otherwise. They later presented a complete set of experiment results for network settings in multi-channel 802.11 wire-

4. The multi-channel multicast problem formulation

We first construct mathematical programming formulations of the optimal multicast problem in WMNs, with multi-gateways and multi-channels. Envisioning two different physical layer technologies for selecting a frequency band for a channel, we present two corresponding optimization models. The first one is based on flexible frequency bands enabled by variable frequency oscillators, such as assumed in software-defined radios. This ideal radio model leads to optimal multicast throughput that can be computed precisely, through the classic primal–dual optimization framework. The second model is rather similar, but makes a more realistic assumption on frequency bands based on the state-of-the-art IEEE 802.11 standard: each transmission has to use one of the 13 pre-defined channels.

Given the tight coupling of different layers in such networks, joint optimization across layers has attracted great interest [9,20,23]. Alicherry et al. [5] presented a joint orthogonal channel assignment and unicast throughput maximization framework. Rad et al. [23] investigated channel allocation, interface assignment and MAC design altogether. Merlin et al. [21] further provided a joint optimization framework for congestion control, channel allocation, interface binding and scheduling to enhance the throughput of multi-hop wireless meshes. Their framework accommodates different channel assignments, but neighbouring channel interference has yet to be addressed.

Recently, Chiu et al. [9] proposed a joint channel assignment and routing protocol for 802.11-based multi-channel mobile ad hoc networks. While sharing many similarities with wireless meshes, the mobility concern and associated overheads are not critical in mesh networks given that the mesh routers and gateways are generally static.

Our work was motivated by these pioneer studies; yet our focus is mainly on throughput maximization in the multicast context, completing the research done in [15,17]. For multicast routing, Nguyen and Xu [24] systematically compared the conventional minimum spanning trees or shortest path trees in wireless meshes. Novel approaches customized for wireless meshes have also been proposed [28,31,32]. Our work is closely related to two of them. In the work of Zeng et al. [32], two heuristics for multicast channel assignment were proposed, which also applies to a multi-gateway configuration. They however did not explicitly address route optimization. In the work of Yuan et al. [31], routing and wireless medium contention were jointly considered. The impact of link interferences and power amplitude variations on each link were also closely examined, but were limited to single channel usage. Our work differs from them in that we examine both multicast routing and channel assignment in a coherent cross-layer framework, and present effective solutions. We also explicitly explore the potentials of multi-gateway configurations.
4.1. Network model and notations

We model a WMN as a graph $G = (V, E)$, with nodes $V$ and links $E$. Assume $T \subseteq V$ is the set of collaborative gateways. Each gateway has a high-bandwidth connection to the Internet, and can be viewed as a data source. Let $S$ be the set of data transmission sessions. We define five vectors of variables. The first four are: the vector of data flows $f = (f_{ij}|i \in S, e \in E)$; the vector of multicast throughput $r = (ri|e \in E)$; the vector of link bandwidth capacities $c = (c_{ij}|e \in E)$; and the power assignment vector $p = (pa \leq p_{amax}|u \in V)$. The last one is on channel assignment. We assume that each node is equipped with one radio with capacity $b$, which can transmit at different frequencies with adjustable power. $b$ is the frequency bandwidth the radio has available for transmission. In the flexible channel model, we have the vector of centre frequencies $\mu = (\mu_{u}|u \in V)$. The frequency band of the channel used by $u$ is then $[\mu_{u} - b/2, \mu_{u} + b/2]$. In the case of fixed channels, we have the vector $\gamma = (\gamma_{u} \in \Gamma|u \in V)$ to represent the channel assignment at each node. Here $\Gamma$ represents the set of pre-defined channels, such as the 13 in the IEEE 802.11 standard. Table 1 summarizes the variables used in the formulation.

4.2. The flexible channel model

Two capacity regions are fundamental to our multicast problem formulation: the channel region and the routing region, at the MAC/PHY layer and the network layer, respectively. The channel region $H$ defines a set of $(c, h)$ such that channel assignment in $h$ can support link capacity vector $c$. The routing region $R$ defines a set of $(r, f)$ such that the throughput vector $r$ can be supported by flow rates in $f$. Detailed characterization of the two regions are not immediately relevant to the overall optimization structure, and are postponed to Sections 6.1 and 6.4 respectively, where we select optimal solutions from each region. The multicast throughput for each session is measured as the data flow rates at the receivers, which are equal for receivers across the same session. A basic physical rule that establishes a connection between the routing region and the channel region is that the aggregated data flow rates have to be bounded by the corresponding link capacities. Furthermore, we follow the convention [31] in modelling throughput utility, and adopt the concave utility function $\log(1 + ri)$ for session throughput $ri$. Then, the throughput maximization problem can be formulated as:

Maximize $U(r) = \sum_{i \in S} U(ri) = \sum_{i \in S} \log(1 + ri)$

Subject to $(c, \mu, p) \in H$

$r, f \in R$

$\sum_{i \in S} ri \leq c, \forall e \in E$

The first constraint $(c, \mu, p) \in H$ models the dependence of effective channel bandwidth on channel assignment and power assignment at each node. The second constraint $(r, f) \in R$ models the dependence of multicast throughput $r$ on the routing scheme $f$. $\sum_{i \in S} ri \leq c, \forall e \in E$ model link capacity constraints. The objective function $U(r)$ is concave, and both the routing and channel regions are convex regions. Therefore, convex optimization methods [7] can be used to compute the optimal solution $(\mu^*, p^*, f^*)$. In Sections 5 and 6, we present a primal–dual solution based on Lagrange relaxation and iterative primal–dual optimization.

If nodes can transmit using pre-defined channels only, we can modify the mathematical program in (1), by replacing the frequency vector $\mu$ with the channel assignment vector $\gamma$. Since $\gamma$ is an integer vector, the mathematical program can not be directly solved to optimal using conventional convex optimization methods, in polynomial time. Nonetheless, the solutions in Sections 5 and 6 will be flexible enough to compute approximate solutions, based on a heuristic channel assignment method.

5. The primal–dual solution framework

The overall solution framework we propose for solving (1) is an iterative primal–dual schema, which switches between solving primal sub-problems and updating dual variables. We describe in Section 5.1 how to decompose the primal problem while introducing dual variables, and then present the primal–dual solution framework in Section 5.2.

5.1. The routing vs. channel assignment decomposition

A critical observation of the optimization problem (1) is that, the channel region $H$ and the routing region $R$ characterize variables from the MAC/PHY layer and the network layer respectively, and are relatively independent. The only coupling constraint between them is $f \ll c$. We can apply the

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Variable</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_{se}$</td>
<td>Gain associated with link $e$</td>
<td>$\phi$</td>
<td>Interference factor = $\sum_{i \in S} I_{ri}/d_{ij}$</td>
</tr>
<tr>
<td>$p_{l}$</td>
<td>Power on link $e$</td>
<td>$\Gamma(v)$</td>
<td>Set of channel activated on node $v$</td>
</tr>
<tr>
<td>$\sigma^{2}$</td>
<td>Noise on line $e$</td>
<td>$\Gamma$</td>
<td>Set of all available channels</td>
</tr>
<tr>
<td>$I_{l}$</td>
<td>Correlation coefficient from $i$ on $e$</td>
<td>$\sigma$</td>
<td>Multicast rate from source $s$</td>
</tr>
<tr>
<td>$G_{se}$</td>
<td>Interference coefficient from $i$ on $e$</td>
<td>$G_{i}$</td>
<td>Set of all gateways in the network</td>
</tr>
<tr>
<td>$x$</td>
<td>Dual variable vector</td>
<td>$G_{i}$</td>
<td>Flow on link $e$ from source $s$</td>
</tr>
<tr>
<td>$c_{l}$</td>
<td>Capacity on link $e$</td>
<td>$e_{ij}$</td>
<td>Conceptual flow on link $e$ from $s$ to $j$</td>
</tr>
<tr>
<td>$b$</td>
<td>Bandwidth</td>
<td>$O(v)$</td>
<td>Set of outgoing links from node $v$</td>
</tr>
<tr>
<td>$p_{lmax}$</td>
<td>Total power budget on node $v$</td>
<td>$l(v)$</td>
<td>Set of incoming links on node $v$</td>
</tr>
</tbody>
</table>
Lagrange relaxation technique [7,19] to remove \( f < c \) from the constraint set, and add a corresponding price term into the objective function: 

\[
L = U(r) + \sum_{e \in E} [c_e - \sum_{i \in S} f_{i,e}] \nu_e
\]

Here \( \nu_e \) is a vector of Lagrange multipliers, which can be viewed as prices governing the link capacity supply - the larger \( \nu_e \) is, the tighter bandwidth supply at link \( e \) is. After the relaxation, the resulting optimization problem is naturally decomposed into two smaller, easier-to-solve sub-problems, including the Channel Assignment Sub-problem at the MAC/PHY layer:

Maximize \[ \sum_{e \in E} \nu_e c_e \]  
Subject to \((e, \gamma, p) \in H\)

and the Routing Sub-problem at the network layer:

Maximize \[ U(r) - \sum_{e \in E} (\nu_e \sum_{i \in S} f_{i,e}) \]  
Subject to \((r, f) \in R\)

It is interesting to observe that, given a link \( e \) with high price \( \nu_e \), the routing sub-problem will automatically attempt to reduce the amount of flow \( f_{i,e} \) through \( e \) during the next round, since its objective function implies minimizing \( \sum_{e \in E} \nu_e \sum_{i \in S} f_{i,e} \). On the other hand, the channel assignment sub-problem will automatically attempt to create more capacity for \( e \), since its objective function is to maximize \( \sum_{e \in E} \nu_e c_e \).

5.2. The primal–dual solution schema

The primal–dual approach iteratively updates the primal \((f, \mu, p)\) and dual \((\nu)\) solutions. During each iteration, we solve the two sub-problems given the current dual vector \( \nu \), and subsequently update \( \nu \) with the newly computed primal vectors as below. Here \( t \) is the round number, and \( \beta \) is the step size vector.

i. Set \( t = 1 \); initialize \( \nu(0), \) e.g., set \( \nu(0) = 0, \forall e \in E \)

ii. Solve primal sub-problems (2) and (3).

iii. Update the dual domain variables as below:

\[
\nu(t) = \max \left( 0, \left[ \nu(t-1) + \beta(t) \left( \sum_{e \in E} \sum_{i \in S} f_{i,e} - c_e \right) \right] \right)
\]

iv. Set \( t = t + 1 \) and return to step ii, until convergence.

The primal–dual algorithm above converges to an optimum primal solution \((f^*, \mu^*, p^*)\) of the optimization problem (1), as long as the regions \( R \) and \( H \) are convex and the step sizes \( \beta(t) \) are appropriately chosen.

The constraint \( f < c \) is linear, the objective function in (1) is strictly concave. The convexity of the capacity regions \( R \) and \( H \) then ensures that the update in the dual domain (iii) is a sub-gradient for the dual variables in \( \nu \). Therefore as long as the step sizes are appropriately chosen, the dual update converges [19,31]. Strong duality further assures that the convergence point of the primal–dual algorithm corresponds to a global optimum of the network optimization problem in (1).

\[
\beta(t) \geq 0, \lim_{t \to \infty} \beta(t) = 0. \quad \text{and} \quad \sum_{t=1}^{\infty} \beta(t) = \infty. \quad \text{A simple sequence that satisfies the conditions above, is} \quad \beta(t) = a/(mk + n), \quad \text{for some positive constants} \quad a, m \quad \text{and} \quad n.
\]

6. Solving channel assignment and routing sub-problems

In order to obtain a complete solution under the primal–dual schema, we need to design algorithms for solving each of the two primal sub-problems. We next discuss how to solve the channel assignment sub-problem in Sections 6.1 and 6.2, and the routing sub-problem in Section 6.4.

6.1. The channel assignment sub-problem

We now construct a detailed model for the channel capacity region \( H \), and discuss how the resulting channel assignment problem from (2) can be solved. The effective link bandwidth capacity are determined by the signal-to-noise-and-interference ratio (SINR) of the transmission; following the Gaussian channel capacity model [31]:

\[
c_e = \log_2(1 + \text{SINR}_e), \quad \text{SINR}_e = \frac{G_{ce}P_e}{(\sum_{i \in E \neq e} I_{le} \cdot P_i \cdot G_{ie}) + \sigma^2}
\]

Here \( G_{ce} \), \( P_e \) and \( \sigma^2 \) are gain, power and noise associated with a link respectively. \( G_{ie} \) and \( \sigma^2 \) denote the interference coefficient and noise from link \( l \) to link \( e \) respectively. \( I_{le} \) is the channel correlation coefficient, which depends on the separation between channels used by \( l \) and \( e \), e.g., the separation between channels 1 and 4 is 3.

The correlation \( I_{y/y'} \) between two channels \( y \) and \( y' \) are known for all possible channel separations, as shown in Fig. 2 [4]. From this figure, the correlation between any two channels, either flexibly selected or predefined, can be found. For example, for the 13 IEEE 802.11 channels, \( I_{y=7/y=2} = 1.0, I_{y=7/y=3} = 0.7906, I_{y=7/y=4} = 0.5267 \), and \( I_{y=7/y=5} = 0 \). Furthermore, we assume the total budget at each node \( v \) is \( P_{\max} \) and \( O(v) \) is the set of outgoing links from node \( v \). Then, the channel assignment problem for capacity maximization can be formulated as:

\[
\max \sum_{e \in E} \nu_e c_e
\]

Subject to \((e, \gamma, p) \in H\)

\[
I_{le} \geq \theta_{le}, \quad \forall l, e \in E \quad \text{and} \quad \nu_e \geq 0
\]

\[
\nu_e \geq 0
\]

\[
I_{le} \geq \theta_{le}, \quad \forall l, e \in E \quad \text{and} \quad \nu_e \geq 0
\]

Fig. 2. Power leakage for neighbouring 802.11g channels.
Maximize $\sum_{e \in E} \alpha_e c_e$
subject to $c_e = \log_2(1 + SINR_e)$, $\forall e \in E$

$$SINR_e = \frac{G_{ee} P_e}{(\sum_{j \in I} G_{ij} P_j + \sigma^2)}$$

$\forall e \in E$

$$\sum_{e \in \partial(v)} p_e \leq P_{v,\text{max}}, \forall v \in V$$

$\gamma_e \in \Gamma, \forall e \in E$

Without the discrete variables in $\gamma$, (4) can be solved using known techniques, such as using geometric programming or through a power control game [31]. The new challenge in (4) is to compute good channels for vector $\gamma$. In Section 6.2, we present a heuristic solution for efficiently solving $\gamma$, and evaluate its performance later in Section 7.

6.2. Heuristic channel assignment algorithm

We design a heuristic channel assignment algorithm based on the interference factor $\phi_e = \sum_{\gamma \in \Gamma} d_{\gamma e}$ for a given candidate channel $\gamma$. Here $d_{\gamma e}$ is the distance to the nearest node transmitting at channel $\gamma$. In measurement-based systems, $d_{\gamma e}$ is not necessary because the node can sense if a channel is in use in the range of this node and signal strength could be used instead. In coordination-based systems, $d_{\gamma e}$ could be found based on the coordination information. We assume $d_{\gamma e} = \infty$ if channel $\gamma$ is not in use in the network. Our heuristic solution, Algorithm 1, performs a breadth-first-traversal of the WMN. At each node, candidate channels are sorted by the interference factor to already assigned channels at other nodes. Diff erent options are possible in selecting the channel. A greedy algorithm selects the channel $\gamma$ with the smallest $\phi_e$ value, i.e., as apart from neighbouring channels in use as possible. We propose a progressive channel assignment approach instead, and select a channel $\gamma$ with the highest $\phi_e$ below an acceptable threshold $\phi_{th}$. The rational here is to look beyond channel assignment at the current node, and to leave good candidate channels for neighbour nodes.

Algorithm 1. Progressive Channel Assignment

<table>
<thead>
<tr>
<th>Initialization:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma(v) = \emptyset, \forall v \in V$;</td>
</tr>
<tr>
<td>$\phiSet = \emptyset$;</td>
</tr>
</tbody>
</table>

for all $v \in V$ do |

for all $\gamma \in \Gamma$ do |

Compute $\phi_e$; |

if $\phi_e \leq \phi_{th}$ then |

$\phiSet \leftarrow \phiSet \cup \phi_e$; |

end if |

end for all |

if $\phiSet = \emptyset$ then |

Choose $\gamma_e$ with smallest $\phi_e$, and activate it on node $v$: $\Gamma(v) = \gamma_e$; |

else |

Choose $\gamma_e$ from $\phiSet$ with largest $\phi_e$, and activate it on node $v$: $\Gamma(v) = \gamma_e$; |

$\phiSet \leftarrow \emptyset$; |

end if |

end for all |

end for all |

Algorithm 1 consists of a double loop. The outer loop iterates through nodes in the network, and the inner loop iterates through all possible channels. The number of channels is 13 and therefore the total number of iterations is $13|V|$.

6.3. Power adjustment algorithm

In the channel assignment heuristic, we assumed that all nodes use the same power levels to activate their links. In this section, we provide an algorithm to incorporate power adjustment to our channel assignment algorithm.

We propose a power adjustment algorithm based on $\alpha_e$ values computed in the dual domain. When $\alpha_e < 0$ for the power adjustment algorithm, we use the $\alpha_e$ values before resetting the negative $\alpha$ values to zero). Each channel $e$ has its capacity higher than its assigned flow rate from the routing sub-module. For nodes with such outgoing links, we find their interfering nodes, and adjust their power. The adjustment is conducted based on the $\alpha$ value of outgoing links at each node. Therefore, interfering nodes with negative $\alpha$ values will decrease their power, while those with positive $\alpha$ values will increase their link activation power.

Algorithm 2. Power Adjustment Algorithm

<table>
<thead>
<tr>
<th>Initialization:</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_{avg} = \sum_{e \in E} \alpha_e /</td>
</tr>
<tr>
<td>$P_{avg} = \sum_{e \in V} P_e /</td>
</tr>
<tr>
<td>for all $u \in V$ do</td>
</tr>
<tr>
<td>if $\alpha_{O(u)} &lt; 0$ then</td>
</tr>
<tr>
<td>for all $v \in NI(u)$ do</td>
</tr>
<tr>
<td>$P_e = P_e + \frac{\alpha_{O(u)}}{\alpha_{avg}} \times P_{avg}$;</td>
</tr>
<tr>
<td>end for all</td>
</tr>
<tr>
<td>end if</td>
</tr>
<tr>
<td>end for all</td>
</tr>
</tbody>
</table>

The main part of Algorithm 2 consists of a double loop. The outer loop iterates through nodes in the network, and the inner loop iterates through its interfering neighbours. The number of interfering nodes is most $|V|/4$, therefore the total number of iterations is upper-bounded by $|V|^2/4$. The initialization parts consists of two independent loops of $|V|$ and $|E|$ iterations. Therefore, it consists of at most $|V|^2/4 + |V| + |E|$ iterations in total.

Note that since power adjustment on a node may change the interference effect of it on other wireless links, channel assignments of a network may change during the next iteration of the primal–dual algorithm based on power adjustment updates.

6.4. The routing sub-problem

The multicast flow routing problem at the network layer has been extensively studied in the literature during the past decade. Two classes of solutions have been proposed. The first class includes multicast tree based solutions. Since achieving optimal multicast throughput using multicast trees corresponds to the NP-hard problem of Steiner tree packing, one needs to resort to efficient approximation algorithms, such as the KMB algorithm. The second class includes network coding based solutions. By assuming information coding capabilities for nodes in the network, the complexity of the optimal multicast problem decreases from NP-hard to polynomial time solvable [19]. In particular, conceptual flow based linear programming models have been successfully developed for multicast in
various network models [19,31]. In this section, we apply similar techniques and formulate our routing sub-problem into a convex program with all-linear constraints, which can be solved using general convex optimization algorithms such as the interior-point algorithm [7], or tailored subgradient algorithms [19]. We model flows from each of the gateways to different destinations as conceptual flows that do not compete for link bandwidth. \( e_l^j \) denotes the conceptual flow rate on link \( l \) in \( j \)th multicast session to its \( j \)th destination. This assumption, brings network coding assumption into the problem and helps us achieve polynomial time solvable problem. \( R(v) \) is set of incoming links to node \( v \) and \( O(v) \) is the set of outgoing links from node \( v \). The multi-gateway multicast routing sub-problem with network coding can be stated as a convex optimization problem.

Maximize \[ U(r) - \sum_{e \in E} \sum_{s \in G} f_s^e \]
subject to \[ r^j \leq \sum_{l \in (j)} e_l^j, \forall s, \forall j \in V \]
\[ e_l^j \leq f_l^j, \forall s \in G, \forall j \in V, \forall l \in E \]
\[ \sum_{l \in O(v)} e_l^j = \sum_{l \in I(v)} e_l^j, \forall s \in G, \forall v, j \in V \]
\[ f_l^j \geq 0, e_l^j \geq 0, r^j \geq 0 \]

Each source \( s \in G \) models a gateway in our network. Note that the term source is used as gateway in this part and does not necessarily mean that multicast contents are different. Specifically, we assume that all the gateways transmit the same multicast content. First constraint assures that the multicast rate transmitted from each source \( s \) is less than or equal to the rate of the destination with minimum conceptual multicast flow from that source. Second constraint imposes the maximum flow rate limit. Conceptual flows do not compete for link bandwidth. Therefore, their maximum, rather than sum, will put a constraint on actual flow rate on the links. Third constraint imposes the flow conservation constraint on the problem. In other words, sum of input flows from one source to a node are equal to sum of the output flows of the same source out of that node. For compact LP formulation, the convention of assuming a virtual feedback link from multicast receivers to sources is followed. Finally, the last constraint imposes the lower bounds on each optimization variable.

The solution to this convex optimization problem utilizes multiple gateways in the network, to cooperatively multicast data to each destination. In other words, may or may not receive multicast data from more than one gateway node in the network. It does not mean that all receiver nodes will receive multicast data from all gateways; they may or may not be connected to a certain gateway. This limit is imposed by the capacity feedback from our channel assignment algorithm (update on \( \alpha \) by primal–dual algorithm) or non-multicast traffic flows in the network. The latter could be brought into formulation by an update factor in dual variable.

Note that in wireless mesh networks nodes are not usually idle. In other words, when a node is part of a wireless mesh network, it usually has a traffic to transmit. Therefore, we are not increasing the number of nodes working in the network by involving them in the multicast. This means, involvement of more or less nodes in the multicast will not dramatically change the network interference levels. This is one of the practical reasons that the two problems of routing and channel assignment could be looked upon independently in different layers.

7. Simulation results

We have implemented the primal–dual framework and the sub-problem solutions to examine the performance of the proposed algorithm. We present our performance analysis in two simulation scenarios. In the first part we present the throughput and dual variable convergence analysis for a simple network. In the second set of simulations we analyze the primal–dual solution in different combination of nodes and network density, discuss scalability of the solution, and compare it to other routing and channel assignment algorithms for multi-gateway multi-channel wireless mesh networks. We used CVX [12], a package for specifying and solving convex programs [12,13], to solve the routing sub-problem.

7.1. Convergence analysis

To clearly understand the algorithm, the evolve of the multicast rate, shadow price and network capacity are shown in Figs. 3–6, for the network in Fig. 1. We have 25 nodes, in a 5 × 5 two dimensional mesh network. Each line in Fig. 3 represents the dual variable over a single link in the network. The initial power levels for each node in the network is considered as 100mW. All simulation settings are the same for Figs. 3–6. The only difference is, node placement is confined within a smaller space in Figs. 4 and 6. The distance between each two nodes is considered to be 35–45 m in the first and 20–25 m in the second set (dense network). Therefore, although the connectivity is the same the convergence is different in the two scenarios. This is signal quality and interference levels are different in the two scenarios. Signal quality from neighbouring nodes is lower in the second set, but the interference is also lower. Therefore, we do not have a major change in the throughput. But this leads to minor changes in the convergence curve. It is worth to note that node placement is semi-random where each node should be placed in a deterministic topology configuration shown in Fig. 1 and randomly within the range given from the neighbouring nodes. More specifically, we placed each node within \( X \) metres random\( (5) \) metres, where \( X \) is 22.5 in the first and 32.5 in the second set of figures. Both sets are considered in indoor environments with transmission range of 45 m and interference range of 135 m for each node.

These experiments show that topology and interference will not drastically affect the convergence rate of the
problem. Our experiments show that initial careful selection of initial $x$ values and step size in updating the dual variable is more important in the convergence of algorithm.

7.2. Performance analysis

We simulated our algorithm using different number of nodes randomly shaping a wireless mesh network. We first place the nodes randomly in the given area (ranging from $300 \times 300$ m$^2$ to $1000 \times 1000$ m$^2$ in different experiments). Then, we build a two dimensional mesh where each node is connected to at most four neighbours. Simulations are conducted for both indoor and outdoor environments with transmission ranges of 45 and 90 and interference range of 135 and 270 respectively. Each node is equipped with radios running IEEE 802.11 protocol and works with initial transmission power of 100 mW tuned during the simulation. 30% of nodes in each scenario are randomly chosen as receiver nodes and max (4, 10% of nodes) are randomly assigned as gateway nodes. The link capacities are computed using $c_e = \log_2(1 + \text{SINR}_e)$.

In this part of simulations we first compare our channel assignment solution with greedy orthogonal and consecutive channel assignment methods. We decide on channel assignment on each node rather than analytically computing the solution centrally. In other words, the channel assignment is performed in a distributed way.

To be able to compare our solution, we also have implemented two other algorithms: (a) orthogonal channel assignment and (b) consecutive channel assignment. In (a), the 13 802.11 channels are assigned to mesh nodes in a consecutive fashion (from channel 1 to 13, then back to 1), during a BFS traversal. In (b), the greedy approach of selecting a channel with maximum separation is adopted.

Figs. 7 and 8 present the network throughput in a WMN including 6–80 nodes in a $1000 \times 1000$ m$^2$ area. This throughput is the maximum throughput in the whole network and is computed based on maximum link capacities in the network (min cut/max flow). In Fig. 7 a random network is generated for a given number of nodes, then the output of all three algorithms is examined on the given single network. Fig. 7, on the other hand, compares the average throughput obtained by each algorithm over 10 random networks of a given number of nodes. The variance of the number of nodes is intended for observing the performance of the solutions with different levels of interference.

Figs. 9 and 10 present the network throughput in a WMN including 60 nodes in an area of $300 \times 300$ m$^2$ to $1000 \times 1000$ m$^2$. Again, Fig. 9 compares the throughput
over a single random network and Fig. 10 presents the average values over 10 random networks. Shaky behaviour of the curve in Fig. 9 is a result of random node placement in the network which may lead to bottleneck links in a random network. Lower values in smaller areas are because of higher interference and more often collisions. An overall observation in Figs. 7–10 is that our solution leads both orthogonal and consecutive channel assignment methods, with a largest margin of up to 100%. The improvement increases as the network size increases. Note that the throughput of orthogonal channel assignment and our proposed algorithm could be improved by scheduling mechanisms.

Then, we examine the performance of the primal–dual algorithm. Figs. 11 and 14 present the maximum multicast throughput using our algorithm as well as consecutive and orthogonal channel assignment algorithms. The multicast routing is assumed to be hop count-based multi-gateway multicast routing used with orthogonal and consecutive channel assignment algorithms. Again, the results are shown for a random network and average over 10 random networks. Results for the single random network is presented to show that for some networks the results of our algorithm might be the same as the orthogonal channel assignment. This is especially true for lower interference levels. Intuitively, when interference is low and not a serious concern, a judiciously designed multi-channel transmission scheme becomes less important. We conclude that our proposed solution is more beneficial when applied in networks with high transmission activities and high interference.

Over figures Figs. 11–14 we can also discuss the scalability of our algorithm. The performance of the algorithm is affected by the density of the nodes in the network rather than overall number of nodes. Thus, for any large given number of nodes throughput is dependent on the density of the nodes in an area where the interference range of the already included nodes expands. Therefore, the network considered as combination of smaller mesh networks of different densities given in figure Fig. 14 will result in the same throughput with central or distributed solution implemented.

8. Conclusion

Multicast applications that require high throughput have recently gained popularity. We studied in this paper
the challenges in achieving high multicast throughput in wireless mesh networks. Two techniques in system design were assumed: introducing multiple mesh gateways for mitigating the gateway bottleneck problem and utilizing multiple wireless channels for combating wireless interference. Our overall solution framework is a primal–dual schema based on a mathematical programming formulation of the optimal multicast problem. The framework iteratively switches between solving primal sub-problems for channel allocation and routing, and dual variable update, and gradually progresses towards optimal or approximately optimal solutions. We further presented precise models for each primal sub-problem, and discussed solutions for each of them. Simulation results confirmed the proposed solutions, in considerable throughput gains that were observed over straightforward approaches of multi-channel multicast.

References
