Why can't we resolve P vs. NP?

"Barriers:

1. Relativization: black-box techniques
2. Algebrization
3. Natural proofs

1. For oracles A, B:
   \[ P^A \neq NP^A \Rightarrow P \neq NP \]

2. Imp: \( \text{PSPACE} = \text{IP} \)
Doesn't relativize! \( \exists \text{ oracle } C \text{ s.t. } \text{PSPACE} \neq \text{IP} \)

Non-relativizing techniques:
(1) Locality of TM computation

3-SAT is \( \text{NP-complete} \)

(2) Arithmetization:
\[ y(x_1, \ldots, x_n) \mapsto p_y(x_1, \ldots, x_n) \]
where \( y \) is poly of low degree, \( p_y \) agrees.
$11) + (12) \Rightarrow \text{IP} = \text{PSPACE}$

Clique, IS, VC, ...

Algorithm:

Have access to oracle of special form:

$A$ oracle : $A : \{0,1\}^n \rightarrow \{0,1\}^n$

$\bar{A} : \mathbb{Z}^n \rightarrow \mathbb{Z}$

$A = \bar{A}$ on $\{0,1\}^n$

poly extension of $A$
A: self-algebraizing
if \( \tilde{A} \in PA \)