How to find Epipolar Geometry for Stereo Matching?
Wildly studied - typical approaches can be categorized as:
- Statistical Feature Point matching:
  - Find feature points in the image, e.g. Harris Corners.
  - Optimize to find best consistent features generating a fundamental matrix. E.g. RANSAC, DLT, LMEASAC.
- Need many point features and narrow baseline image pair.
- Feature Neighborhood matching:
  - Find feature points in the image, similarly to above.
  - Describe feature neighborhoods, e.g. Affine 2D moment matrix, SIFT.
  - Need many features, affine transforms for wide baseline image pair.
- Projective Invariant features:
  - Find projective invariant features, e.g. Bitangents, Cross-ratio.
  - Need few features but global object outline and assumptions.

PROBLEM: How to recover Epipolar geometry from few features for a wide baseline image pair?

What is a Spatial Topology Graph?

Spatial Feature Graph contains:
- Vertices: Feature points of the image.
- Edges: Edgemap curves sectioned by vertices.
- Order: Edge incidence order for a vertex.
- Probabilities: Edge and vertex existence probabilities.

Spatial Neighborhood Graph contains:
- Vertices: Feature curves, the dual of spatial feature graph edges.
- Edges: Connect image curves that are immediate neighbors.
- Probabilities: Edge and vertex existence probabilities.

Spatial Topology Graphs for Feature-Minimal Correspondence

Analysis of projective invariant and robust features:

<table>
<thead>
<tr>
<th>Feature Graph</th>
<th>Projective Invariant Features</th>
<th>Projective Robust Features</th>
</tr>
</thead>
<tbody>
<tr>
<td>Edges + Neighbor Graph Vertices</td>
<td>Exist in the graphs under projection change, even if they are incomplete in the image (\rightarrow) Matching allowed.</td>
<td>Image curve curvatures change slowly with view change. Curvature mean is largely independent of length.</td>
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<td>Feature Graph Vertices</td>
<td>Exist in the graphs under projection change (\rightarrow) Matching allowed. Vertex degrees must be identical and ordered similarly.</td>
<td>Vertex incidence angles change slowely with view change, similarly to curve curvatures.</td>
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<tr>
<td>Neighborhood Graph Edges</td>
<td>Exist under projection change, even if incomplete (\rightarrow) Matching allowed. Can also handle missing feature edges.</td>
<td>The percentage of length of projection on a curve by a nearest neighbor curve changes slowly, but not resilient to occlusions.</td>
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We avoid defining any features of length \(\rightarrow\) nonlinear shortening.

Feature Extraction:
The image Features are derived from an image edgemap.
The curvature for curve point \(\kappa(u) = (x'(u), y'(u))\) is:
\[
\kappa(u) = \frac{x'y'' - x''y'}{(x'^2 + y'^2)^{3/2}}
\]
Use Curvature Scale Space (CSS) to detect feature points as curvature extrema \(\rightarrow\) Feature graph vertices.
Endpoints and joints where curvature is undefined are vertices as well.

Sample Dinosaur image processing:

Probabilistic Graph Matching:
The probability that any two features correspond is given by:
\[
P(\{f, f'\} \in M) = P(\{f, f'\} \in M \mid f, f')P(f, f') + P(\{f, f'\} \in M \mid f, f')P(f, f')P(f)P(f')
\]
Where edge prior probability \(P(e)\) depends on edge strength, and vertex prior \(P(v)\) depends on extremal ratio over neighborhood and
\[
P(v) = P(v_1, ..., v_n)P(v_1, ..., v_n)
\]
The probability of two edges matching depends on the bending energy:
\[
E_v = \frac{1}{4} \int_{[\gamma_i]} \beta e^{2} da P(v, e') = e^{-\varepsilon_2 \sum_{e_i} E_v(v_i, e_i)}
\]
The probability of two vertices matching depends on vertex angles \(E_v\):
\[
P(v, v') = \exp \left( -\frac{1}{2} \sum_{i=1}^{n} E_v(v_i, e_i) - E_v(v_i, e_{i+1}) \right)
\]
The probability of a set of correspondences is given by:
\[
P(M) = P(M) \prod_{\{f, f'\} \in M} P(\{f, f'\} \in M)
\]
Where the prior probability is higher for larger, more connected sets.

The matching algorithm:
The matching approach is a heuristic tree search. The tree nodes are sets of estimated feature pairs.
- Sort the top n matching feature pairs in decreasing order.
- Repeat at most n times
  - Add the next feature in the sorted feature pair list to all the branches from the most likely to the least.
  - Prune incompatible feature pairs from branch if necessary.
  - Try to add neighboring features, and apply other heuristics.
  - Terminate when next feature pair cannot improve best probability.

Matching results:
The results were obtained for the sample Dinosaur from a sparse data set. Around 100 features (4000 pairs) match in \(< 1\) min on 3GHz PC.

Conclusions:
- The Spatial Topology Graph can reliably match few feature under a general projective change, with resilience to topology change.
- The motion above is large enough to cause significant topology change in some parts. Yet, the most probable matching feature pairs are correct.
- It is beneficial to define topology graph on features' diffuse colors as well.