Evolutionary Equilibria in Computer Networks: Specialization and Niche Formation

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Modelling User Communities

- A system provides users with access to resources, e.g. a network.
- Centralized planning: gather requests, compute optimal allocation.
- “Anarchy”: users individually choose resources, e.g. routes for messages.
- Individual choice → strategic interactions (≈ traffic models).
Central Allocation

Users → Messages → Router → Network

- 500K
- 250K
- 750K

Evolutionary Equilibria in Network Games
Decentralized Individual Choice

Users → Messages → Network

- 500K
- 250K
- 750K
Motivation for Game-Theoretic Modelling

Use game theory to predict outcome of “selfish” user choices (Nash equilibrium)

1. Assess “price of anarchy”
2. Improve network design/protocols
Outline

- Parallel Links Model
- Bayesian Parallel Links Game
- Intro to Evolutionary Stability
- ESS for Parallel Links Game
  - Characterization
  - Structural Conditions
Parallel Links Model

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<thead>
<tr>
<th>Tasks</th>
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<tr>
<td>100K</td>
<td>500K</td>
<td>250K</td>
<td>750K</td>
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<th>Speeds</th>
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<td>10</td>
<td>20</td>
<td>15</td>
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delay of task $w$ on link $l = \frac{w}{\text{speed of } l}$

Evolutionary Equilibria in Network Games
Parallel Links Model as a Game
(Koutsoupias and Papadimitriou 1999)

1. Players 1,..,n with tasks \(w_1, \ldots, w_n\)
2. Pure strategy = (choice of) link
3. Fix choices \((w_1,l_1), \ldots, (w_n,l_n)\).
   \[\Rightarrow \text{load on link } l = \sum_{i=1}^{n} w_i \text{ for } l_i = l.\]
   \[\Rightarrow \text{utility } u_i \text{ for player } i = \]
   \[\text{- load on link } l_i\]
   \[\text{speed of link } l_i\]
Bayesian Routing Game
(Gairing, Monien, Tiemann 2005)

• Agents are uncertain about tasks.
  • common dist. $\mu$ over tasks $W$
  • strategy $\sim$ "program" $p$ for routing tasks
  • $p(l|w)$ = probability that program $p$ chooses link $l$ when given task $w$.

• $u_i(p_1,\ldots,p_n) = \sum_{\text{task assignments } <w_1,\ldots,w_n>} \prod_{j=1..n} \mu(w_j) \cdot u_i [(w_1,p_1|w_1),\ldots,(w_n,p_n|w_n)]$
Motivation for Evolutionary Analysis

1. Under “anarchy”, we expect successful strategies to spread → evolutionary dynamics.

2. Highly successful predictions in biology.

3. Distinguishes stable from unstable equilibria.

4. May be useful in network design:
   see W. Sandholm’s (2002) pricing scheme for traffic congestion. “evolutionary implementation in computer networks seems an important topic for future research”.

Evolutionary Equilibria in Network Games
Hawk vs. Dove As A Population Game

<table>
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<tr>
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<th>Hawk (H)</th>
<th>Dove (D)</th>
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<tr>
<td>Hawk</td>
<td>-2, -2</td>
<td>6, 0</td>
</tr>
<tr>
<td>Dove</td>
<td>0, 6</td>
<td>3, 3</td>
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- Assume a large population of agents.
- Agents are either hawks (H) or doves (D).
- We randomly draw 2 at a time to play.
Population Interpretation of Nash Equilibrium

1. Consider a population of agents with frequency distribution $\pi$. 
   e.g. [H,H,H,H,H,H,D,D,D,D]

2. $\pi$ is in equilibrium  
   $\Leftrightarrow$ H does as well as D  
   $\Leftrightarrow$ $(\pi, \pi)$ is a symmetric Nash equilibrium.

3. $(\pi, \pi)$ does not represent the choices of 2 players.

4. $(\pi, \pi)$ says that both positions are drawn from the same population of agents with distribution $\pi$. 
Stable vs. Unstable Equilibrium

unstable

stable within a neighbourhood
Evolutionarily Stable Strategies (ESS)

mixed population dist. = (1-\(\varepsilon\)) \(\pi^*\) + \(\varepsilon\pi\)

<table>
<thead>
<tr>
<th>current dist (\pi^*)</th>
<th>mutant dist (\pi)</th>
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<tr>
<td>HHHHHH DDDD</td>
<td>H D</td>
</tr>
<tr>
<td>10/12 = 1-(\varepsilon)</td>
<td>2/12 = (\varepsilon)</td>
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1. A distribution \(\pi^*\) is an ESS if and only if for all sufficiently small mutations \(\pi\) the incumbents in \(\pi^*\) do better in the mixed population than the mutants.

2. A distribution \(\pi^*\) is an ESS if and only if there is an \(\varepsilon^*\) such that for all sizes \(\varepsilon < \varepsilon^*\) \(u(\pi^*; (1-\varepsilon) \pi^* + \varepsilon\pi) > u(\pi; (1-\varepsilon) \pi^* + \varepsilon\pi)\) for all mutations \(\pi \neq \pi^*\).
Characterization of ESS in Bayesian Routing Game $B$

Define:

- the load on link $l$ due to strategy $p$:
  $$\text{load}(p,l) = \sum_{tasks} w \mu(w) \cdot p(l|w) \cdot w$$

- the (marginal) probability of using link $l$:
  $$\text{prob}(p,l) = \sum_{tasks} w \mu(w) \cdot p(l|w)$$

**Theorem.** A strategy $p^*$ is an ESS in $B \iff$ for all best replies $p \neq p^*$ we have

$$\sum_{\text{links}} \left[ \text{load}(p^*,l)-\text{load}(p,l) \right] \cdot \left[ \text{prob}(p^*,l)-\text{prob}(p,l) \right] > 0$$

Intuition: to defeat mutation $p$:

- if load on link increases, use link less (- x -)
- if load decreases, use link more (+ x +)

Evolutionary Equilibria in Network Games
Proposition. Let $B$ be a Bayesian routing routing game with ESS $p^*$. If two links $l_1, l_2$ have the same speed, then $p^*(l_1|w) = p^*(l_2|w)$ for all tasks $w$. 

<table>
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<th>Links</th>
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<tr>
<td>$w_1:50%, w_2:50%, w_3:0$</td>
<td>10</td>
</tr>
<tr>
<td>$w_1:50%, w_2:50%, w_3:0$</td>
<td>10</td>
</tr>
<tr>
<td>$w_1:0, w_2:0, w_3:100%$</td>
<td>15</td>
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Necessary Condition: bigger tasks get faster links

**Proposition.** Let $B$ be a Bayesian routing game with ESS $p^*$. Suppose that

1. link 1 is faster than link 2
2. $p^*$ uses link 1 for task $w_1$, link 2 for task $w_2$.

Then $w_1 \geq w_2$.

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<tr>
<td>$w_2$ = 10: 50%</td>
<td>10</td>
</tr>
<tr>
<td>$w_1$ = 20:100%, $w_2$: 50%</td>
<td>15</td>
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Single Task: Unique ESS

**Proposition.** Let $B$ be a Bayesian network routing game with just one task $w$.

1. $B$ has a unique ESS $p^*$.

2. If all $m$ links have the same speed, $p^*(l_j | w) = 1/m$ is the unique ESS.

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<td>$w$: 1/3</td>
<td>10</td>
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<td>$w$: 1/3</td>
<td>10</td>
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<td>$w$: 1/3</td>
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Strong Necessary Condition: No Double Overlap

- Fix a Bayesian network game $B$.
- Strategy $p^*$ **uses** link $l$ for weight $w$ $\iff$ $p^*(l|w) > 0$.

**Proposition.** Let $p^*$ be an ESS in $B$. Suppose that $p^*$ uses two distinct links $l_1 \neq l_2$ for task $w$. Then $p^*$ does not use both $l_1$ and $l_2$ for any other task $w'$.

<table>
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<tr>
<td>$w_1$:70%, $w_2$:30%</td>
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<tr>
<td>$w_1$:30%, $w_2$:70%</td>
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<tr>
<td>$w_3$:100%</td>
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</table>
Proposition. Let $B$ be a Bayesian network game with $>1$ link, $>1$ task, all links the same speed. Then there is no ESS for $B$. 

$$
\begin{array}{c|c|c}
\text{Links} & \text{Speeds} \\
\hline
\text{double overlap} & \text{50\%} & \text{50\%} & \text{10} \\
\text{50\%} & \text{50\%} & \text{10} \\
\end{array}
$$
Clusterings are typical ESS’s

- Fix a Bayesian network game $B$ with strategy $p^*$.
- A link $l$ is **optimal** for task $w$ given $p^*$ ⇔ $l$ minimizes $w$/speed($l$) + load($l$, $p^*$).
- A strategy $p^*$ **clusters** ⇔ if two distinct links $l_1 \neq l_2$ are optimal for task $w$, then neither $l_1$ nor $l_2$ is optimal for any other task $w' \neq w$.

**Proposition.** If $p^*$ clusters, then $p^*$ is an ESS.
Does A Clustered Equilibrium Exist?

- Fix an assignment $A$ of links to tasks.

**Proposition.**

1. There is \textit{at most one} clustered ESS $p^*$ whose clustering is $A$.
2. The candidate $p^*$ can be computed in polynomial time.
3. The question: is there a clustered ESS $p^*$ for a game $B$? is in NP.
Future Work

- Conjecture: if an ESS exists, it’s unique.
- Conjecture: the “no double overlap” condition is sufficient as well as necessary.
- Computational Complexity and Algorithms for computing ESS’s.
Conclusion

- ESS *refines* Nash equilibrium and defines *stable* equilibria.
- Analysis of evolutionary stability in Bayesian network games:
  - characterization of *successful mutations*
  - *structure* of stable *task/link allocations*.
- Finding:
  - evolutionary dynamics leads to formation of “niches” or *clusters* for task/link combinations.
  - Symmetric outcomes tend to be socially suboptimal.