CMPT409/815: Advanced Algorithms

Homework Assignment 1

Instructor: Igor Shinkar *Due date: September 24, 2019*

Instructions: Submit either in class (hard copy) or to Coursys (if scanned, make sure it's good quality).

Question 1 (25 points)

(a) Prove that any graph G on n vertices has at most $n(n-1)/2$ cuts of smallest size.

(b) Prove that the upper bound of $n(n-1)/2$ on the number min cuts is tight.

Hint: (a) Recall Karger's algorithm we saw in class. (b) Consider the cycle graph on n vertices.

Question 2 (25 points) *Denote by $\alpha(G)$ the size of the maximum independent set in G.*

Design an algorithm that given a graph G that contains an independent set of size $\alpha(G) \geq (\frac{1}{2}+\delta)|V|$, outputs an independent set of size $\geq 2\delta|V|$.

Hint: Prove that G contains a vertex cover of size k if and only if it contains an independent set of size $|V|-k$.

Question 3 (25 points) *An r-uniform hypergraph $H=(V,E)$ is a collection of vertices V, and a collection of hyperedges E, where each hyperedge $e \in E$ is a subset of V of size $|e|=r$. The case of $r=2$ corresponds to graphs.*

Consider the Minimum Vertex Cover *problem for r-uniform hypergraphs: Given an r-uniform hypergraph the goal is to find a collection of vertices $C \subseteq V$ of minimum size such that for every $e \in E$ it holds that $e \cap C \neq \emptyset$. Design a r-approximation algorithm for the problem of finding a minimum vertex cover in an r-uniform hypergraph.*

Question 4 (25 points) *Design a* deterministic *algorithm that gets a graph $G=(V,E)$ and outputs a 3-coloring $C: V \to \{RED, BLUE, GREEN\}$ of the vertices such that for at least $2|E|/3$ edges their endpoints receive different colors.*