CMPT409/815: Advanced Algorithms

Homework Assignment 2

Instructor: Igor Shinkar *Due date: October 8, 2019*

Instructions: Submit either in class (hard copy) or to Coursys (if scanned, make sure it's good quality).

Question 1 (25 points) *Consider the setting where we have n bins, and n balls are placed into the bins independently, each ball is chooses a bin uniformly.*

(a) Prove that with probability at least $1 - 1/n$ no bin contains more than $O(\lg(n)/\log\log(n))$ balls.

(b) Prove that with constant probability at least $n/10$ bins remain empty.

Question 2 (25 points) *Design an polynomial time algorithm for the Set-Cover problem with the following guarantee. The input is a universe U of size n, sets $S_1, S_2, \ldots, S_m \subseteq U$, and a parameter k such that there are k sets that cover U. The algorithm outputs a collection of t sets that cover U such that $t \leq k + k\ln(n/k)$.*

Question 3 (25 points) *Design a polynomial time algorithm that gets as input a graph G on n vertices, that is guaranteed to have a clique of size $n/log^{10}(n)$. The output of the algorithm is a clique in G of size $\log(n)/\log\log(n)$.*

Question 4 (25 points) *Design a polynomial time algorithm whose input is a (not necessarily bipartite) graph $G = (V, E)$ that contains a perfect matching, and the output is a perfect matching in G.*

Instructions: Consider the $|V \times |V|$ matrix Z defined as

$$Z(i,j) = \begin{cases} X_{i,j} & \text{if } (i,j) \in E \text{ and } i < j \\ -X_{j,i} & \text{if } (i,j) \in E \text{ and } j < i \\ 0 & \text{otherwise} \end{cases},$$

where $X_{i,j}$ are formal variables. Prove that $det(Z)$ is identically zero if and only if G does not contain a perfect matching. Use this fact to output a perfect matching in G.