CMPT409/815: Advanced Algorithms

Homework Assignment 3

Instructor: Igor Shinkar *Due date: October 29, 2019*

Instructions: Submit either in class (hard copy) or to Coursys (if scanned, make sure it is good quality).

Question 1 (25 points) *In the* UNIQUE-CLIQUE *problem the input is a graph $G = (V, E)$ and $k \in \mathbb{N}$. An algorithm is said to solve the* UNIQUE-CLIQUE *problem if it satisfies the following guarantees.*

YES case *: If G has a clique of size k and the maximum size clique is unique, the algorithm outputs YES.*

NO case *: If G has no clique of size k, the algorithm must output NO.*

Remark*: If G has a clique of size at least k, and has more than one clique of maximum size, the algorithm may output anything.*

Show a randomized reduction from the MAX-CLIQUE *problem to the* UNIQUE-CLIQUE *problem. That is, show a randomized polynomial time reduction (an algorithm) that gets a graph H and a parameter $k' \in \mathbb{N}$, and outputs a graph G and a parameter k and satisfies the following guarantees.*

YES case *: If H has a clique of size at least k', then G has a clique of size at least k, and has a* unique *clique of maximum size with probability at least 0.9.*

NO case *: If H has no clique of size k', then G has no clique of size k.*

Conclude that if UNIQUE-CLIQUE *can be solved in polynomial time, then the* MAX-CLIQUE *problem can be solved using a poly-time randomized algorithm.*

(Hint: Use the isolation lemma.)

Question 2 (25 points) *Solve the following linear problem. Prove that your solution is optimal by writing the dual LP, and showing a corresponding feasible solution for the dual LP.*

$$\text{minimize}_{x,y,z\in\mathbb{R}} \quad 3x + 5y + 2z$$

$$\text{subject to} \begin{cases} x + 6y & \geq 1 \\ 2y + 2z & \geq 2 \\ x - 2y + 3z & \geq 2 \\ 2x - 2y + z & \geq 3 \\ x, y, z \geq 0 \end{cases}$$

Question 3 (25 points)

(a) Write an integer linear programming (ILP) formulation for the 3-coloring problem. (There is more than one way to write such an ILP. You may choose any formulation)

(b) Relax the ILP to LP, and show a graph that is not 3-colorable that has a feasible solution to your LP. (Note that this is a feasibility problem, and not an optimization problem)

(c) Write the Dual LP for the linear program in the previous item.

Question 4 (25 points) *In the max-cut problem the input is a graph $G = (V, E)$, and the goal is to find a subset $S \subseteq V$ that maximizes $|\{(u, v) \in E : u \in S, v \notin S\}|$.*

(a) Write a CSP formulation for the MAX-CUT problem.

(b) Write the level-2 Sherali-Adams LP for the MAX-CUT problem. Specify explicitly the variables, the constraints, and the objective function.