CMPT409/815: Advanced Algorithms

Homework Assignment 5

Instructor: Igor Shinkar *Due date: December 2, 2019*

Instructions: Submit either in class (hard copy) or to Coursys (if scanned, make sure it is good quality).

Question 1 (25 points) *Prove that* $\mathsf{PCP}_{1,0.5}[r = O(\log(n)), q = \text{poly}(n)] = \mathsf{NP}$.

Question 2 (25 points) *Let* $s \in (0,1)$, *and* $r, q \in \mathbb{N}$. *Prove that*

$$\mathsf{PCP}_{1,s}[r,q]_{\{0,1\}} \subseteq \mathsf{PCP}_{1,s'}[r + \lceil \log(q) \rceil, 2]_{\Sigma=\{0,1\}^q},$$

where $s' = 1 - \frac{1-s}{q}$.

Question 3 (25 points) *DEG-2-SAT is the following problem. The input is a finite field* $\mathbb{F}$ *(think of* $\mathbb{F} = \mathbb{F}_2$, *and a collection of polynomials* $p_1, \ldots, p_m : \mathbb{F}^n \to \mathbb{F}$ *of degree at most 2. In the search version of the problem the goal is to find an assignment* $a = (a_1, \ldots, a_n) \in \mathbb{F}^n$ *such that* $p_i(a) = 0$ *for all* $i \in [m]$. *In the decision version of the problem the goal is to decide whether such an assignment exists.*

Prove that $DEG-2-SAT$ *is* NP*-complete.*

Question 4 (25 points) *For parameters* $0 < s < c < 1$ *consider the gap-DEG-2-SAT*$_{c,s}$ *problem. The input is a finite field* $\mathbb{F}$, *and a collection of polynomials* $p_1, \ldots, p_m : \mathbb{F}^n \to \mathbb{F}$ *of degree at most 2. In the decision version of the problem the goal is to decide between*

1. *(YES case): there exists an assignment* $a = (a_1, \ldots, a_n) \in \mathbb{F}^n$ *such that* $|\{i \in [m] : p_i(a) = 0\}| \geq cm$.
2. *(NO case): every assignment* $a = (a_1, \ldots, a_n) \in \mathbb{F}^n$ *such that* $|\{i \in [m] : p_i(a) = 0\}| < sm$.

In the search version of the problem we are guaranteed that there exists an assignment $a = (a_1, \ldots, a_n) \in \mathbb{F}^n$ *such that* $|\{i \in [m] : p_i(a) = 0\}| \geq cm$, *and the goal is to find an assignment* $a = (a_1, \ldots, a_n) \in \mathbb{F}^n$ *that satisfies* $p_i(a) = 0$ *for more than* sm *polynomials (i.e., disprove the NO case).*

Prove that gap-DEG-2-SAT$_{1,2/3}$ *is* NP*-complete.*

(Hint: Use the following fact: **Explicit linear error correcting codes:** *For any* $m \in \mathbb{N}$ *there exists* $m' \leq \text{poly}(m)$ *and a matrix* $M \in \mathbb{F}^{m \times m'}$ *with the following guarantee: For any non-zero* $x \in \mathbb{F}^m$ *the vector* $y = Mx \in \mathbb{F}^{m'}$ *satisfies* $|\{i \in [m'] : y_i \neq 0\}| \geq m'/3$. *Furthermore, such* M *can be constructed in time* $\text{poly}(m)$.