CMPT881: Special Topics in Theoretical Computer Science

Homework assignment 2

Instructor: Igor Shinkar *Due date: March 7, 2019*

Question 1 *Given a (non-bipartite) graph $G = (V, E)$ with n vertices, consider the following $n \times n$ skew-symmetric matrix A_G.*

$$A_G(i,j) = \begin{cases} X_{i,j} & \text{if } (i,j) \in E, i < j \\ -X_{j,i} & \text{if } (i,j) \in E, j < i \\ 0 & \text{otherwise} \end{cases}$$

- *Prove that the determinant of A_G is non-zero (as a polynomial) if and only if G has a perfect matching.*
- *Design a randomized polynomial time algorithm that given a graph G with probability > 0.9 outputs a perfect matching in G or reports that G does not have a perfect matching.*

Question 2 *Given $b, c, d \in \mathbb{R}^n$ and $A \in \mathbb{R}^{n \times n}$ consider the following minimization problem.*

$$\begin{aligned} \text{minimize}_{x \in \mathbb{R}^n} \quad & \tfrac{c^T x}{d^t x} \\ \text{subject to :} \quad & \\ & Ax \leq b \\ & d^T x \geq 2 \\ & c^T x \geq 4 \end{aligned}$$

Show how to find a solution that is within 0.01 from the optimum in polynomial time.

Hint: cast the problem into an LP

Question 3 *Solve the following linear problem. Prove that your solution is optimal by writing the dual LP, and showing a corresponding feasible solution for the dual*

$$\begin{aligned} \text{minimize}_{x,y,z \in \mathbb{R}} \quad & 5x + 3y - 4z \\ \text{subject to :} \quad & \\ & x + 6y \geq 1 \\ & 2y + 4z \geq 2 \\ & 2x - 2y - 3z \geq 3 \\ & x, y, z \geq 0 \end{aligned}$$

Question 4 *Write an integer linear programming (ILP) formulation for the 3-coloring problem. (There is more than one way to write such an ILP. You may choose any formulation)*

Relax the ILP to LP, and show a graph that is not 3-colorable that has a feasible solution to your LP. (Note that this is a feasbility problem, and not an optimization problem)