Lecture Outline

- **Binary**
  - **Decimal, Binary, and Hexadecimal**
  - **Base Conversion**
  - **Binary Encoding**

Decimal Numbering System

- Ten symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9

- Represent larger numbers as a sequence of digits
  - Each digit is one of the available symbols

- Example: 7061 in decimal (base 10)
  - $7061_{10} = (7 \times 10^3) + (0 \times 10^2) + (6 \times 10^1) + (1 \times 10^0)$

Octal Numbering System

- Eight symbols: 0, 1, 2, 3, 4, 5, 6, 7
  - Notice that we no longer use 8 or 9
- Base comparison:
  - Base 10: `0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12…`
  - Base 8: `0, 1, 2, 3, 4, 5, 6, 7, 10, 11, 12, 13, 14…`

- Example: What is 7061_8 in base 10?
  - $7061_8 = (7 \times 8^3) + (0 \times 8^2) + (6 \times 8^1) + (1 \times 8^0) = 3633_{10}$

Warmup Question

- What is 34_8 in base 10?

A. 32_{10}
B. 34_{10}
C. 7_{10}
D. 28_{10}
E. 35_{10}

Binary and Hexadecimal

- Binary is base 2
  - Symbols: 0, 1
  - Convention: $2_{10} = 10_2 = 0b10$
- Example: What is 0b110 in base 10?
  - $0b110 = 110_2 = (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) = 6_{10}$
- Hexadecimal (hex, for short) is base 16
  - Symbols? 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, **A, B, C, D, E, F**
  - Convention: $16_{10} = 10_{16} = 0x10$
- Example: What is 0xA5 in base 10?
  - $0xA5 = A5_{16} = (10 \times 16^1) + (5 \times 16^0) = 165_{10}$

Peer Instruction Question

- Which of the following orderings is correct?

A. 0xC < 0b1010 < 11

B. 0xC < 11 < 0b1010

C. 11 < 0b1010 < 0xC

D. 0b1010 < 11 < 0xC

E. 0b1010 < 0xC < 11

Converting to Base 10

- Can convert from any base *to* base 10
  - $0b110 = 110_2 = (1 \times 2^2) + (1 \times 2^1) + (0 \times 2^0) = 6_{10}$
  - $0xA5 = A5_{16} = (10 \times 16^1) + (5 \times 16^0) = 165_{10}$

- We learned to think in base 10, so this is fairly natural for us

- **Challenge:** Convert into other bases (*e.g.* 2, 16)

Challenge Question

- Convert 13_{10} into binary

- Hints:
  - $2^3 = 8$
  - $2^2 = 4$
  - $2^1 = 2$
  - $2^0 = 1$

Converting from Decimal to Binary

- Given a decimal number N:
  - List increasing powers of 2 from *right to left* until $\geq$ N
  - Then from *left to right*, ask is that (power of 2) $\leq$ N?
    - If **YES**, put a 1 below and subtract that power from N
    - If **NO**, put a 0 below and keep going

- Example: 13 to binary

2^4=16	2^3=8	2^2=4	2^1=2	2^0=1

Converting from Decimal to Base B

- Given a decimal number N:
  - List increasing powers of B from *right to left* until $\geq$ N
  - Then from *left to right*, ask is that (power of B) $\leq$ N?
    - If **YES**, put *how many* of that power go into N and subtract from N
    - If **NO**, put a 0 below and keep going

- Example: 165 to hex

16^2=256	16^1=16	16^0=1

Converting Binary ↔ Hexadecimal

- Hex → Binary
  - Substitute hex digits, then drop any leading zeros
  - Example: 0x2D to binary
    - 0x2 is 0b0010, 0xD is 0b1101
    - Drop two leading zeros, answer is 0b101101
- Binary → Hex
  - Pad with leading zeros until multiple of 4, then substitute each group of 4
  - Example: 0b101101
    - Pad to 0b 0010 1101
    - Substitute to get 0x2D

Base 10	Base 2	Base 16
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Binary → Hex Practice

- Convert 0b1001101101101101
  - How many digits?
  - Pad:
  - Substitute:

Base 10	Base 2	Base 16
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Base Comparison

- Why does all of this matter?
  - *Humans* think about numbers in **base 10**, but *computers* "think" about numbers in **base 2**
  - Binary encoding is what allows computers to do all of the amazing things that they do!
- You should have this table memorized by the end of the class
  - Might as well start now!

Base 10	Base 2	Base 16
0	0000	0
1	0001	1
2	0010	2
3	0011	3
4	0100	4
5	0101	5
6	0110	6
7	0111	7
8	1000	8
9	1001	9
10	1010	A
11	1011	B
12	1100	C
13	1101	D
14	1110	E
15	1111	F

Numerical Encoding

- **AMAZING FACT: You can represent *anything* countable using numbers!**
  - Need to agree on an encoding
  - Kind of like learning a new language

- Examples:
  - Decimal Integers: 0→0b0, 1→0b1, 2→0b10, etc.
  - English Letters: CSE→0x435345, yay→0x796179
  - Emoticons: 😀 0x0, 😞 0x1, 😎 0x2, 😇 0x3, 😈 0x4, 🤷 0x5

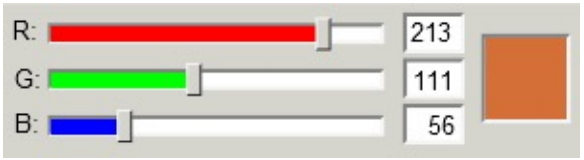
Binary Encoding

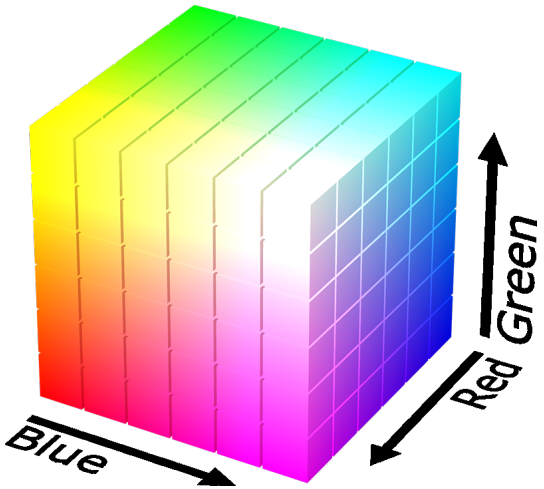
- With N binary digits, how many "things" can you represent?
  - Need N binary digits to represent n things, where $2^N \geq n$
  - Example: 5 binary digits for alphabet because $2^5 = 32 > 26$

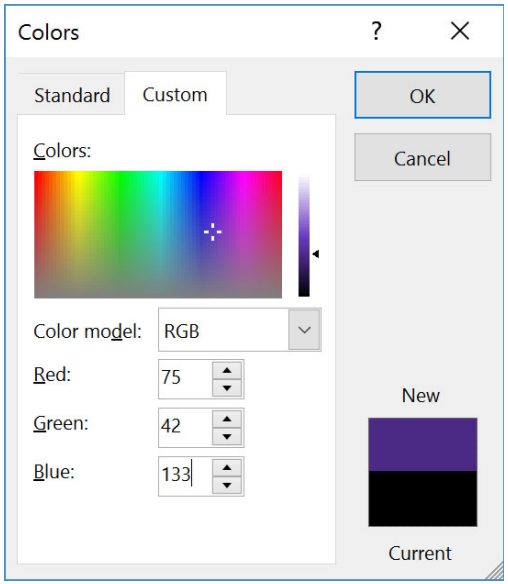
- A binary digit is known as a bit
- A group of 4 bits (1 hex digit) is called a nibble
- A group of 8 bits (2 hex digits) is called a byte
  - 1 bit → 2 things, 1 nibble → 16 things, 1 byte → 256 things

Binary Encoding – Colors

- RGB – Red, Green, Blue
  - Additive color model (light): byte (8 bits) for each color
  - Commonly seen in hex (in HTML, photo editing, etc.)
  - Examples: **Blue**→0x0000FF, **Gold**→0xFFD700, **White**→0xFFFFFF, **Deep Pink**→0xFF1493







Binary Encoding – Characters/Text

- ASCII Encoding (www.asciitable.com)
  - American Standard Code for Information Interchange

Dec	Hx	Oct	Char	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr	Dec	Hx	Oct	Html	Chr
0	0	000	NUL (null)	32	20	040		Space	64	40	100	@	@	96	60	140	`	`
1	1	001	SOH (start of heading)	33	21	041	!	!	65	41	101	A	A	97	61	141	a	a
2	2	002	STX (start of text)	34	22	042	"	"	66	42	102	B	B	98	62	142	b	b
3	3	003	ETX (end of text)	35	23	043	#	#	67	43	103	C	C	99	63	143	c	c
4	4	004	EOT (end of transmission)	36	24	044	$	$	68	44	104	D	D	100	64	144	d	d
5	5	005	ENQ (enquiry)	37	25	045	%	%	69	45	105	E	E	101	65	145	e	e
6	6	006	ACK (acknowledge)	38	26	046	&	&	70	46	106	F	F	102	66	146	f	f
7	7	007	BEL (bell)	39	27	047	'	'	71	47	107	G	G	103	67	147	g	g
8	8	010	BS (backspace)	40	28	050	(	(	72	48	110	H	H	104	68	150	h	h
9	9	011	TAB (horizontal tab)	41	29	051	)	)	73	49	111	I	I	105	69	151	i	i
10	A	012	LF (NL line feed, new line)	42	2A	052	*	*	74	4A	112	J	J	106	6A	152	j	j
11	B	013	VT (vertical tab)	43	2B	053	+	+	75	4B	113	K	K	107	6B	153	k	k
12	C	014	FF (NP form feed, new page)	44	2C	054	,	,	76	4C	114	L	L	108	6C	154	l	l
13	D	015	CR (carriage return)	45	2D	055	-	-	77	4D	115	M	M	109	6D	155	m	m
14	E	016	SO (shift out)	46	2E	056	.	.	78	4E	116	N	N	110	6E	156	n	n
15	F	017	SI (shift in)	47	2F	057	/	/	79	4F	117	O	O	111	6F	157	o	o
16	10	020	DLE (data link escape)	48	30	060	0	0	80	50	120	P	P	112	70	160	p	p
17	11	021	DC1 (device control 1)	49	31	061	1	1	81	51	121	Q	Q	113	71	161	q	q
18	12	022	DC2 (device control 2)	50	32	062	2	2	82	52	122	R	R	114	72	162	r	r
19	13	023	DC3 (device control 3)	51	33	063	3	3	83	53	123	S	S	115	73	163	s	s
20	14	024	DC4 (device control 4)	52	34	064	4	4	84	54	124	T	T	116	74	164	t	t
21	15	025	NAK (negative acknowledge)	53	35	065	5	5	85	55	125	U	U	117	75	165	u	u
22	16	026	SYN (synchronous idle)	54	36	066	6	6	86	56	126	V	V	118	76	166	v	v
23	17	027	ETB (end of trans. block)	55	37	067	7	7	87	57	127	W	W	119	77	167	w	w
24	18	030	CAN (cancel)	56	38	070	8	8	88	58	130	X	X	120	78	170	x	x
25	19	031	EM (end of medium)	57	39	071	9	9	89	59	131	Y	Y	121	79	171	y	y
26	1A	032	SUB (substitute)	58	3A	072	:	:	90	5A	132	Z	Z	122	7A	172	z	z
27	1B	033	ESC (escape)	59	3B	073	;	;	91	5B	133	[	[	123	7B	173	{	{
28	1C	034	FS (file separator)	60	3C	074	<	<	92	5C	134	\	\	124	7C	174			|
29	1D	035	GS (group separator)	61	3D	075	=	=	93	5D	135	]	]	125	7D	175	}	}
30	1E	036	RS (record separator)	62	3E	076	>	>	94	5E	136	^	^	126	7E	176	~	~
31	1F	037	US (unit separator)	63	3F	077	?	?	95	5F	137	_	_	127	7F	177	&#127;	DEL

Source: www.LookupTables.com

Binary Encoding – Files and Programs

- At the lowest level, all digital data is stored as bits!

- Layers of abstraction keep everything comprehensible
  - Data/files are groups of bits interpreted by program
  - Program is actually groups of bits being interpreted by your CPU

Summary

- Humans think about numbers in decimal; computers think about numbers in binary
  - Base conversion to go between them
  - Hexadecimal is more human-readable than binary
- All information on a computer is binary
- Binary encoding can represent *anything*!
  - Computer/program needs to know how to interpret the bits