Locality, Communication, and Contention
Warm up:
communication using message passing
(since it makes communication explicit)
Recall the grid-based solver example

In previous lectures we expressed this parallel program using data parallel and SPMD programming abstractions

```c
int N;
float* A = allocate(n+2, n+2);

void solve(float* A) {
    bool done = false;
    float diff = 0.f;
    while (!done) {
        for_all (red cells (i,j)) {
            float prev = A[i,j];
            reduceAdd(diff, abs(A[i,j] - prev));
        }
        if (diff/(N*N) < TOLERANCE)
            done = true;
    }
}
```
Let’s think about expressing a parallel grid solver with communication via messages.

One possible message passing machine configuration: a cluster of two workstations.
Review: message passing model

- Threads operate within their own private address spaces
- Threads communicate by sending/receiving messages
  - `send`: specifies recipient, buffer to be transmitted, and optional message identifier ("tag")
  - `receive`: sender, specifies buffer to store data, and optional message identifier
  - Sending messages is the only way to exchange data between threads 1 and 2
    - Why?

Illustration adopted from Culler, Singh, Gupta
Message passing model: each thread operates in its own address space

In this figure: four threads

The grid data is partitioned into four allocations, each residing in one of the four unique thread address spaces (four per-thread private arrays)
Data replication is now required to correctly execute the program

Example:
After processing of red cells is complete, thread 1 and thread 3 send one row of data to thread 2 (thread 2 requires up-to-date red cell information to update black cells in the next phase)

“Ghost cells” are grid cells replicated from a remote address space. It’s common to say that information in ghost cells is “owned” by other threads.

Thread 2 logic:

```c
float* local_data = allocate(N+2,rows_per_thread+2);
int tid = get_thread_id();
int bytes = sizeof(float) * (N+2);

// receive ghost row cells (white dots)
recv(&local_data[0,0], bytes, tid-1);
recv(&local_data[rows_per_thread+1,0], bytes, tid+1);
```

// Thread 2 now has data necessary to perform // future computation
```
int N;
int tid = get_thread_id();
int rows_per_thread = N / get_num_threads();

float* localA = allocate(rows_per_thread+2, N+2);

// assume localA is initialized with starting values
// assume MSG_ID_ROW, MSG_ID_DONE, MSG_ID_DIFF are constants used as msg ids

void solve() {
    bool done = false;
    while (!done) {
        float my_diff = 0.0f;

        if (tid != 0)
            send(&localA[1,0], sizeof(float)*(N+2), tid-1, MSG_ID_ROW);
        if (tid != get_num_threads()-1)
            send(&localA[rows_per_thread,0], sizeof(float)*(N+2), tid+1, MSG_ID_ROW);

        if (tid != 0)
            recv(&localA[0,0], sizeof(float)*(N+2), tid-1, MSG_ID_ROW);
        if (tid != get_num_threads()-1)
            recv(&localA[rows_per_thread+1,0], sizeof(float)*(N+2), tid+1, MSG_ID_ROW);

        for (int i=1; i<rows_per_thread+1; i++) {
            for (int j=1; j<n+1; j++) {
                float prev = localA[i,j];
                                    localA[i,j-1] + localA[i,j+1]);
                my_diff += fabs(localA[i,j] - prev);
            }
        }

        if (tid != 0) {
            send(&mydiff, sizeof(float), 0, MSG_ID_DIFF);
            recv(&done, sizeof(bool), 0, MSG_ID_DONE);
        } else {
            float remote_diff;
            for (int i=1; i<get_num_threads()-1; i++)
                recv(&remote_diff, sizeof(float), i, MSG_ID_DIFF); my_diff += remote_diff;
        }

        if (my_diff/(N*N) < TOLERANCE)
            done = true;
        for (int i=1; i<get_num_threads()-1; i++)
            send(&done, sizeof(bool), i, MSG_ID_DONE);
    }
}
```

Message passing solver

Similar structure to shared address space solver, but now communication is explicit in message sends and receives.

Send and receive ghost rows to “neighbor threads”

Perform computation
(just like in shared address space version of solver)

All threads send local my_diff to thread 0

Thread 0 computes global diff, evaluates termination predicate and sends result back to all other threads

Example pseudocode from: Culler, Singh, and Gupta
Notes on message passing example

- **Computation**
  - Array indexing is relative to local address space

- **Communication:**
  - Performed by sending and receiving messages
  - Bulk transfer: communicate entire rows at a time

- **Synchronization:**
  - Performed by sending and receiving messages
  - Consider how to implement mutual exclusion, barriers, flags using messages
Synchronous (blocking) send and receive

- `send()`: call returns when sender receives acknowledgement that message data resides in address space of receiver

- `recv()`: call returns when data from received message is copied into address space of receiver and acknowledgement sent back to sender
As implemented on the prior slide, there is a big problem with our message passing solver if it uses synchronous send/recv!

Why?

How can we fix it?
(while still using synchronous send/recv)
Message passing solver (fixed to avoid deadlock)

Send and receive ghost rows to “neighbor threads”
Even-numbered threads send, then receive
Odd-numbered thread recv, then send

Example pseudocode from: Culler, Singh, and Gupta
Non-blocking asynchronous send/recv

- **send():** call returns immediately
  - Buffer provided to send() cannot be modified by calling thread since message processing occurs concurrently with thread execution
  - Calling thread can perform other work while waiting for message to be sent

- **recv():** posts intent to receive in the future, returns immediately
  - Use checksend(), checkrecv() to determine actual status of send/receipt
  - Calling thread can perform other work while waiting for message to be received

<table>
<thead>
<tr>
<th>Sender:</th>
<th>Receiver:</th>
</tr>
</thead>
<tbody>
<tr>
<td>Call SEND(foo)</td>
<td>Call RECV(bar)</td>
</tr>
<tr>
<td>SEND returns handle h1</td>
<td>RECV(bar) returns handle h2</td>
</tr>
<tr>
<td>Copy data from ‘foo’ into network buffer</td>
<td></td>
</tr>
<tr>
<td>Send message</td>
<td>Receive message</td>
</tr>
<tr>
<td>Call CHECKSEND(h1) //if message sent, now safe for thread to modify ‘foo’</td>
<td>Messaging library copies data into ‘bar’</td>
</tr>
</tbody>
</table>

RED TEXT = executes concurrently with application thread
When I talk about communication, I’m not just referring to messages between machines. (e.g., in a datacenter)

More examples:

Communication between cores on a chip
Communication between a core and its cache
Communication between a core and memory
Think of a parallel system as an extended memory hierarchy

I want you to think of “communication” very generally:
- Communication between a processor and its cache
- Communication between processor and memory (e.g., memory on same machine)
- Communication between processor and a remote memory
  (e.g., memory on another node in the cluster, accessed by sending a network message)

View from one processor

Accesses not satisfied in local memory cause communication with next level

So managing locality to reduce the amount of communication performed is important at all levels.
Review: latency vs throughput

Latency

The amount of time needed for an operation to complete.
A memory load that misses the cache has a latency of 200 cycles
A packet takes 20 ms to be sent from my computer to Google
Asking a question on Groups gets a response in 10 minutes

Throughput

The rate at which operations are performed.
Memory can provide data to the processor at 25 GB/sec (memory bandwidth)
A communication link can send 10 million messages per second
The TAs answer 50 questions per day on Piazza
Asimple model of communication

Example: sending a N-bit message

![Diagram: Processor 1 to Processor 2沟通示意图]

Link can communicate B (bits/1 cycle)
Latency of first bit to travel to destination = T cycles

How long does it take for an entire message to be sent from processor 1 to processor 2?

One thought: \( T_0 \times \left( \frac{N}{B} \right) \)
Another thought: \( T_0 + \frac{N}{B} \)

??
Latency vs Throughput

Assume only one car in a lane of the highway at once.

Car’s velocity: 100 km/hr

Distance: ~ 50 km

Latency of driving: 0.5 hours

Throughput: 2 cars per hour
Improving throughput

Approach 1: drive faster!
Throughput = 4 cars per hour

Approach 2: build more lanes!
Throughput = 8 cars per hour (2 cars per hour per lane)
Now caravan of 12 cars allowed on highway at once!

Latency of first car to arrive = 0.5 hrs
Four cars from caravan arrive every 1/100th of an hour
Total time for 12-car caravan: 0.5 hrs + 3/100th of an hour
“Effective car throughput” = 12 cars / 0.53 hours = ~ 23 cars / hour

Message size = N bits
Link can communicate B (bits/clk)
Latency of first bit to travel to destination = T clks
Time to send message = T + N/B clocks
If message must arrive before the link can send the next message, then “Effective bandwidth” = N bits / (T + N/B) clocks
Using the highway more efficiently

Cars spaced out by 1 km

Throughput: 100 cars/hr (1 car every 1/100th of hour)

Throughput: 400 cars/hr (4 cars every 1/100th of hour)
Pipelining
A general model of msg communication

Steps in sending a N-bit message:

- Processor copies data from memory to network card.
- Send data over link 1 (slow link).
- Send data over link 2 (fast link).
- Copy data from network to memory.

The total latency of message send is equal to the occupancy (time for data to pass through the slowest component of the system).
Pipelined communication of messages

Assume processor copies messages into buffer for network to transmit from. Buffer is fixed size and can hold at most two messages (numbers indicate number of msgs in buffer after insert).

Sender generates msgs at full rate until output buffer fills (faster than 1/occupancy).

Sender stalls because output buffer is full.

Occupancy determines communication rate!

In steady state throughput is $\propto \frac{1}{\text{occupancy}}$ msg/sec.
Another example: CPU to memory communication

Processor issues load instruction

Send request to memory

Cache lookup

Transfer cache line from memory over memory bus

Transfer value to processor register

Time to send cache line over memory bus

total latency of memory access
Example: memory bound instruction stream!

Cache misses result in request added to size=1 memory request buffer

Notice: memory fully utilized, but processor’s utilization drop to rate determined by BW

Question: what happens to processor utilization if the ratio of math instructions to load instructions is increased?
Good questions about the previous slide

- How do I see from the figure that the memory bus is fully utilized?
- How would I illustrate higher memory latency (keep in mind memory requests are pipelined and memory bus bandwidth is not changed)?
- How would the figure change if memory bus bandwidth was increased?
- Would there be processor stalls if the ratio of math instructions to load instructions was significantly increased? Why?
Communication-to-computation ratio

\[
\text{amount of communication (e.g., bytes)} \quad \text{amount of computation (e.g., instructions)}
\]

- If denominator is the execution time of computation, ratio gives average bandwidth requirement of code

- \textit{“Arithmetic intensity”} = \(1 /\text{communication-to-computation ratio}\)
  - I find arithmetic intensity a more intuitive quantity, since higher is better.
  - It also sounds cooler

- High arithmetic intensity (low communication-to-computation ratio) is required to efficiently utilize modern parallel processors since the ratio of compute capability to available bandwidth is high (recall element-wise vector multiply example from the end of lecture 2)
Two reasons for communication: inherent vs. artifactual communication
Inherent communication

Communication that must occur in a parallel algorithm. The communication is fundamental to the algorithm.

In our messaging passing example at the start of class, sending ghost rows was inherent communication.
Reducing inherent communication

Good assignment decisions can reduce inherent communication (increase arithmetic intensity)

1D blocked assignment: $N \times N$ grid

1D interleaved assignment: $N \times N$ grid

Elements computed (per processor) $\approx \frac{N^2}{P}$

$\propto \frac{N}{P}$

Elements communicated (per processor) $\approx 2N$

Elements computed

Elements communicated

= $1/2$
Reducing inherent communication

2D blocked assignment: Nx Ngrid

\[ N^2 \text{ elements} \]

\[ \frac{N^2}{P} \]

\[ P \text{ processors} \]

\[ \propto \frac{N}{\sqrt{P}} \]

\[ \frac{N}{\sqrt{P}} \]

Asymptotically better communication scaling than 1D blocked assignment

Communication costs increase sub-linearly with \( P \)

Assignment captures 2D locality of algorithm
Artifactual communication

- Inherent communication: information that fundamentally must be moved between processors to carry out the algorithm given the specified assignment (assumes unlimited capacity caches, minimum granularity transfers, etc.)

- Artifactual communication: all other communication (artifactual communication results from practical details of system implementation)
Data access in grid solver: row-major traversal

Assume row-major grid layout.
Assume cache line is 4 grid elements.
Cache capacity is 24 grid elements (6 lines)

Recall grid solver application.
Blue elements show data that is in cache after update to red element.
Assume row-major grid layout.
Assume cache line is 4 grid elements.
Cache capacity is 24 grid elements (6 lines)

Blue elements show data in cache at end of processing first row.
Problem with row-major traversal: long time between accesses to same data

Assume row-major grid layout.
Assume cache line is 4 grid elements.
Cache capacity is 24 grid elements (6 lines)

Although elements (0,2) and (0,1) had been accessed previously, they are no longer present in cache at start of processing row 2.

This program loads three lines for every four elements of output.
Artifactual communication examples

- System might have a minimum granularity of data transfer (result: system must communicate more data than what is needed)
  - Program loads one 4-byte float value but entire 64-byte cache line must be transferred from memory (16x more communication than necessary)

- System operation might result in unnecessary communication:
  - Program stores 16 consecutive 4-byte float values, and as a result the entire 64-byte cache line is loaded from memory, entirely overwritten, then subsequently stored to memory (2x overhead... load was unnecessary)

- Finite replication capacity (the same data communicated to processor multiple times because cache is too small to retain it between accesses)
Techniques for reducing communication
Improving temporal locality by changing grid traversal order

Assume row-major grid layout.
Assume cache line is 4 grid elements.
Cache capacity is 24 grid elements (6 lines)

“Blocked” iteration order

(diagram shows state of cache after finishing work from first row of first block)

Now load two cache lines for every six elements of output
Improving temporal locality by fusing loops

void add(int n, float* A, float* B, float* C) {
    for (int i=0; i<n; i++)
        C[i] = A[i] + B[i];
}
void mul(int n, float* A, float* B, float* C) {
    for (int i=0; i<n; i++)
        C[i] = A[i] * B[i];
}
// assume arrays are allocated here
// compute E = D + ((A + B) * C)
    add(n, A, B, tmp1);
    add(n, tmp1, C, tmp2);
    add(n, tmp2, D, E);

void fused(int n, float* A, float* B, float* C, float* D, float* E) {
    for (int i=0; i<n; i++)
        E[i] = D[i] + (A[i] + B[i]) * C[i];
}
// compute E = D + (A + B) * C
    fused(n, A, B, C, D, E);

Two loads, one store per math op (arithmetic intensity = 1/3)
Two loads, one store per math op (arithmetic intensity = 1/3)
Overall arithmetic intensity = 1/3

Code on top is more modular (e.g., array-based math library like numPy in Python)
Code on bottom performs much better. Why?
Improve arithmetic intensity by sharing data

- Exploit sharing: co-locate tasks that operate on the same data
  - Schedule threads working on the same data structure at the same time on the same processor
  - Reduces inherent communication
Exploiting spatial locality

- High granularity communication (e.g., a cache line) may introduce artifactual communication
  - If application has low spatial locality, system may transfer data that program did not need
Artifactual communication due to comm. granularity

Consider 2D blocked assignment of data to processors described previously. Assume: communication granularity is a cache line, and a cache line contains four elements.

Inherently need one element from left and right neighbor, but system must communicate four.

Implication: artifactual communication increases with cache line size!

= required elements assigned to other processors
Artifactual communication due to cache line communication granularity

Data partitioned in half by column. Partitions assigned to threads running on P1 and P2

Threads access their assigned elements (no inherent communication exists)

But data access on real machine triggers (artifactual) communication due to the cache line being written to by both processors *

*Further detail in the upcoming cache coherence lectures
Reducing artifactual comm: blocked data layout

(Blue lines indicate consecutive memory addresses)

2D, row-major array layout

4D array layout (block-major)

Consecutive addresses straddle partition boundary

Consecutive addresses remain within single partition

Note: don’t confuse blocked assignment of work to threads (true in both cases above) with blocked data layout of data elements in the address space (only at right)
Observations:
- Static assignment is sufficient (approximately equal busy time per thread)
- 4D blocking of grid reduces time spent on communication (reflected on graph as data wait time)
- Synchronization cost is largely due to waiting at barriers
Contention
Contention

- A resource can perform operations at a given throughput (number of transactions per unit time)
  - Memory, communication links, servers, TA’s at office hours, etc.

- Contention occurs when many requests to a resource are made within a small window of time (the resource is a “hot spot”)

Example: updating a shared variable

Flat communication:
potential for high contention
(but low latency if no contention)

Tree structured communication:
reduces contention
(but higher latency under no contention)
Example: distributed work queues reduce contention

(contention in access to single shared work queue)

Subproblems
(a.k.a. “tasks”, “work to do”)

Set of work queues
(In general, one per worker thread)

Worker threads:
Pull data from own work queue
Push new work to own work queue
(no contention when all processors have work to do)

When local work queue is empty... STEAL work from random work queue
(synchronization okay since processor would have sat idle anyway)
Summary: reducing communication costs

- Reduce overhead of communication to sender/receiver
  - Send fewer messages, make messages larger (amortize overhead)
  - Coalesce many small messages into large ones

- Reduce latency of communication
  - Application writer: restructure code to exploit locality
  - Hardware implementor: improve communication architecture

- Reduce contention
  - Replicate contended resources (e.g., local copies, fine-grained locks)
  - Stagger access to contended resources

- Increase communication/computation overlap
  - Application writer: use asynchronous messages
  - HW implementor: pipelining, multi-threading, pre-fetching,
  - Requires additional concurrency in application
Roofline model

- Use microbenchmarks to compute peak performance of a machine as a function of arithmetic intensity of application
- Then compare application’s performance to known peak values

Figure credit: Williams et al. 2009
Roofline model: optimization regions

- Use various levels of optimization in benchmarks (e.g., best performance with and without using SIMD instructions)

Figure credit: Williams et al. 2009
Establishing high watermarks *

Add “math” (non-memory instructions)
Does execution time increase linearly with operation count as math is added?
(If so, this is evidence that code is instruction-rate limited)

Remove almost all math, but load same data
How much does execution-time decrease? If not much, suspect memory bottleneck

Change all array accesses to A[0]
How much faster does your code get?
(This establishes an upper bound on benefit of improving locality of data access)

Remove all atomic operations or locks
How much faster does your code get? (provided it still does approximately the same amount of work)
(This establishes an upper bound on benefit of reducing sync overhead.)

* Computation, memory access, and synchronization are almost never perfectly overlapped. As a result, overall performance will rarely be dictated entirely by compute or by bandwidth or by sync. Even so, the sensitivity of performance change to the above program modifications can be a good indication of dominant costs.
Use profilers/performance monitoring tools

- Image at left is “CPU usage” from activity monitor in OSX while browsing the web in Chrome (my laptop has a quad-core Core i7 CPU)
  - Graph plots percentage of time OS has scheduled a process thread onto a processor execution context
  - Not very helpful for optimizing performance

- All modern processors have low-level event “performance counters”
  - Registers that count important details such as: instructions completed, clock ticks, L2/L3 cache hits/misses, bytes read from memory controller, etc.

- Example: Intel’s Performance Counter Monitor Tool provides a C++ API for accessing these registers.

```c
PCM *m = PCM::getInstance();
SystemCounterState begin = getSystemCounterState();

// code to analyze goes here

SystemCounterState end = getSystemCounterState();

printf("Instructions per clock: \%f\n", getIPC(begin, end));
printf("L3 cache hit ratio: \%f\n", getL3CacheHitRatio(begin, end));
printf("Bytes read: \%d\n", getBytesReadFromMC(begin, end));
```

- Also see Intel VTune, PAPI, oprofile, etc.
Bonus slides:
Understanding problem size issues can very helpful when assessing program performance
Your boss selects the application that matters most to the company “I want you to demonstrate good performance on this application.”

How do you know if you have a good design?

- **Absolute performance?**
  - Often measured as wall clock time
  - Another example: operations per second

- **Speedup: performance improvement due to parallelism?**
  - Execution time of sequential program / execution time on P processors
  - Operations per second on P processors / operations per second of sequential program

- **Efficiency?**
  - Performance per unit resource
  - e.g., operations per second per chip area, per dollar, per watt
Measuring scaling

- Consider the grid solver example from last week’s class
  - We changed the algorithm to allow for parallelism
  - The new algorithm might converge more slowly, requiring more iterations of the solver

- Should speedup be measured against the performance of a parallel version of a program running on one processor, or the best sequential program?

Common pitfall: compare parallel program speedup to parallel algorithm running on one core (easier to make yourself look good)
Speedup of solver application: 258 x 258 grid

Execution on 32 processor SGI Origin 2000

Figure credit: Culler, Singh, and Gupta
Remember: work assignment in solver

2D blocked assignment: \( N \times N \text{grid} \)

- \( N^2 \) elements
- \( P \) processors

- Elements computed: \( \frac{N^2}{P} \)
  - (per processor)

- Elements communicated: \( \propto \frac{N}{\sqrt{P}} \)
  - (per processor)

Arithmetic intensity: \( \propto \frac{N}{\sqrt{P}} \)

Small \( N \) (or large \( P \)) yields low arithmetic intensity!
Pitfalls of fixed problem size speedup analysis

Solver execution on 32 processor SGI Origin 2000

Problem size is just too small for the machine (large communication-to-computation ratio)

Scaling the performance of small problem may not be all that important anyway (it might already execute fast enough on a single core)

No benefit! (slight slowdown)

258 x 258 grid on 32 processors: ~ 310 grid cells per processor
1K x 1K grid on 32 processors: ~ 32K grid cells per processor

Figure credit: Culler, Singh, and Gupta
Pitfalls of fixed problem size speedup analysis
Execution on 32 processor SGI Origin 2000

Here: super-linear speedup! with enough processors, chunk of grid assigned to each processor begins to fit in cache (key working set fits in per-processor cache)

Another example: if problem size is too large for a single machine, working set may not fit in memory: causing thrashing to disk

(this would make speedup on a bigger parallel machine with more memory look amazing!)

Figure credit: Culler, Singh, and Gupta
Understanding scaling

- There can be complex interactions between the size of the problem to solve and the size of the parallel computer
  - Can impact load balance, overhead, arithmetic intensity, locality of data access
  - Effects can be dramatic and application dependent

- Evaluating a machine with a fixed problem size can be problematic
  - Too small a problem:
    - Parallelism overheads dominate parallelism benefits (may even result in slow downs)
    - Problem size may be appropriate for small machines, but inappropriate for large ones (does not reflect realistic usage of large machine!)
  
  - Too large a problem: (problem size chosen to be appropriate for large machine)
    - Key working set may not “fit” in small machine (causing thrashing to disk, or key working set exceeds cache capacity, or can’t run at all)
    - When problem working set “fits” in a large machine but not small one, super-linear speedups can occur

- Can be desirable to scale problem size as machine sizes grow (buy a bigger machine to compute more, rather than just compute the same problem faster)
Architects also think about scaling

A common question: “Does an architecture scale?”

- **Scaling up:** how does architecture’s performance scale with increasing core count?
  - Will design scale to the high end?

- **Scaling down:** how does architecture’s performance scale with decreasing core count?
  - Will design scale to the low end?

- Parallel architectures are designed to work in a range of contexts
  - Same architecture used for low-end, medium-scale, and high-end systems
  - GPUs are a great example
    - Same SMMcore architecture, different numbers of SMMcores per chip

![Tegra X1](image1.png): 2 SMMcores (mobile SoC)

![GTX950](image2.png): 6 SMMcores (90 watts)

![GTX980](image3.png): 16 SMMcores (165 watts)

![Titan X](image4.png): 24 SMMcores (250 watts)
Questions to ask when scaling a problem

- Under what constraints should the problem be scaled?
  - “Work done by program” may no longer be the quantity that is fixed
  - Fixed data set size, fixed memory usage per processor, fixed execution time, etc.?

- How should the problem be scaled?
  - Problem size is often determined by more than one parameter
  - Solver example: problem defined by \((N, \varepsilon, \Delta t, T)\)

  - Grid size: \((N \times N)\)
  - Total time simulated by program (one hour of fluid flow)
  - Time step size (of overall fluid simulation that uses solver)
  - Convergence threshold of solver
Problem-constrained scaling *

- Focus: use a parallel computer to solve the same problem faster

\[
\text{Speedup} = \frac{\text{time 1 processor}}{\text{time P processors}}
\]

- Recall pitfalls from earlier in lecture (small problems may not be realistic workloads for large machines, big problems may not fit on small machines)

- Examples of problem-constrained scaling:
  - Almost everything we’ve considered parallelizing in class so far

*Problem-constrained scaling is often called “hard scaling”.*
Time-constrained scaling

- **Focus:** completing more work in a fixed amount of time
  - Execution time kept fixed as the machine (and problem) scales

\[
\text{Speedup} = \frac{\text{work done by } P \text{ processors}}{\text{work done by 1 processor}}
\]

- **How to measure “work”?**
  - Challenge: “work done” may not be linear function of problem inputs
    (e.g. matrix multiplication is \(O(N^3)\) work for \(O(N^2)\) sized inputs)
  - One approach: “work done” is defined by execution time of same computation on a single processor (but consider effects of thrashing if problem too big)
  - Ideally, a measure of work is:
    - Simple to understand
    - Scales linearly with sequential run time (so ideal speedup remains linear in \(P\))
Time-constrained scaling example

Real-time 3D graphics: more compute power allows for rendering of much more complex scene

Problem size metrics: number of polygons, texels sampled, shader length, etc.

Image credits:
http://www.game-weavers.com/?page_id=490
Time-constrained scaling example

Large Synoptic Survey Telescope (LSST)

- Acquire high-resolution survey of sky (3-gigapixel image every 15 seconds, every night for many years)

Rapid Image analysis compute platform (detect “potentially” interesting events)

Notify other observatories if potential event detected.

LSST will be located on top of Cerro Pachón Mountain, Chile

Increasing compute capability allows for more sophisticated detection algorithms (fewer false positives, detect broader class of events)

Image credits: [http://www.lsst.org](http://www.lsst.org) [http://mcdonaldobservatory.org](http://mcdonaldobservatory.org)
More time-constrained scaling examples

- **Computational finance**
  - Run most sophisticated model possible in: 1 ms, 1 minute, overnight, etc.

- **Modern websites**
  - Want to generate complex page, respond to user in X milliseconds
    (studies show site usage directly corresponds to page load latency)

- **Real-time computer vision for robotics**
  - Consider self-driving car: want best-quality obstacle detection in 5 ms
Memory-constrained scaling *

- Focus: run the largest problem possible without overflowing main memory **
- Memory per processor is held fixed (e.g., add more machines to cluster)
- Neither work or execution time are held constant

\[
\text{Speedup} = \frac{\text{work (P processors)} \times \text{time (1 processor)}}{\text{time (P processors)} \times \text{work (1 processor)}}
\]

\[
= \frac{\text{work per unit time on P processors}}{\text{work per unit time on 1 processor}}
\]

- Note: scaling problem size can make runtimes very large
  - Consider $O(N^3)$ matrix multiplication on $O(N^2)$ matrices

* Memory-constrained scaling is often called “weak scaling”
** Assumptions: (1) memory resources scale with processor count (2) spilling to disk is infeasible behavior (too slow)
Memory-constrained scaling examples

- One motivation to use supercomputers and large clusters is simply to be able to fit large problems in memory.

- Large N-body problems
  - 2012 Supercomputing Gordon Bell Prize Winner: 1,073,741,824,000 particle N-body simulation on Japan’s K-Computer.

- Large-scale machine learning
  - Billions of clicks, documents, etc.

- Memcached (in memory caching system for web apps)
  - More servers = more available cache.

Image credit: Ishiyama et al. 2012
Scaling examples at PIXAR

- Rendering a “shot” (a sequence of frames) in a movie
  - Goal: minimize time to completion (problem constrained)
  - Assign each frame to a different machine in the cluster

- Artists working to design lighting for a scene
  - Provide interactive frame rate to artist (time constrained)
  - More performance = higher fidelity representation shown to artist in allotted time

- Physical simulation: e.g., fluid simulation
  - Parallelize simulation across multiple machines to fit simulation grid in aggregate memory of processors (memory constrained)

- Final render of images for movie
  - Scene complexity is typically bounded by memory available on farm machines
  - One barrier to exploiting additional parallelism within a machine is that required footprint often increases with number of processors
Summary of tips

- Measure, measure, measure...

- Establish high watermarks for your program
  - Are you compute, synchronization, or bandwidth bound?

- Be aware of scaling issues. Is the problem well matched for the machine?