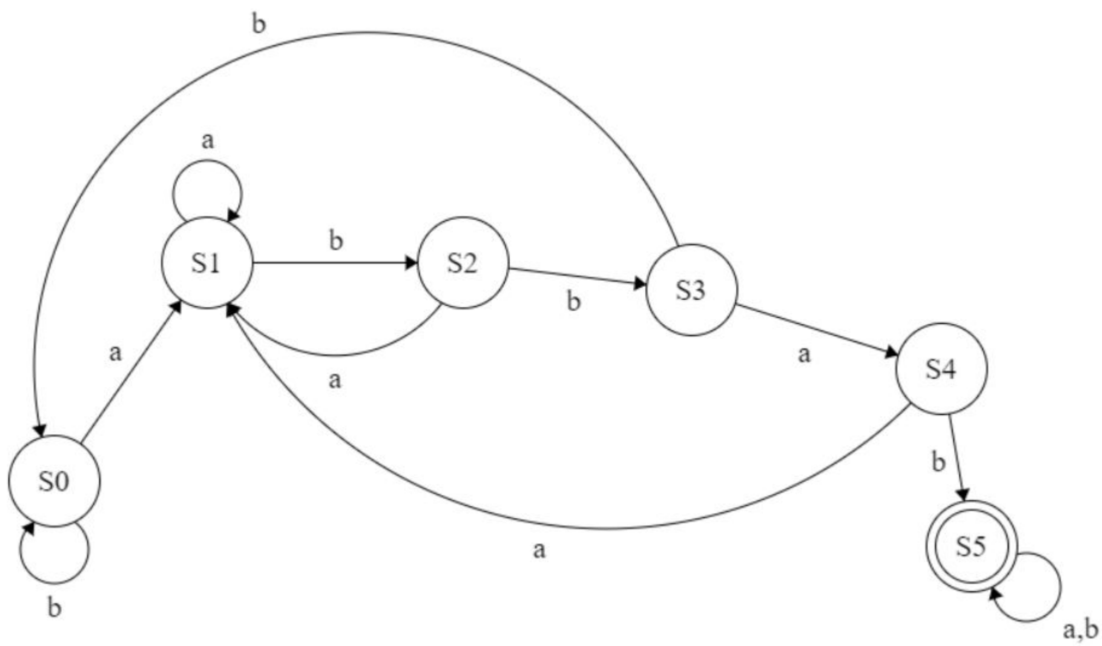
CMPT125, Fall 2018
Homework Assignment 5
Due date: November 30, 2018

Submit homework, printed or written in readable handwriting,
to the assignment boxes in CSIL ASB9838.

1) [20 points] Draw a DFA that defines the language

$$L_1 = \{x \in \{a,b\}^* : x \text{ contains the string abbab}\}$$

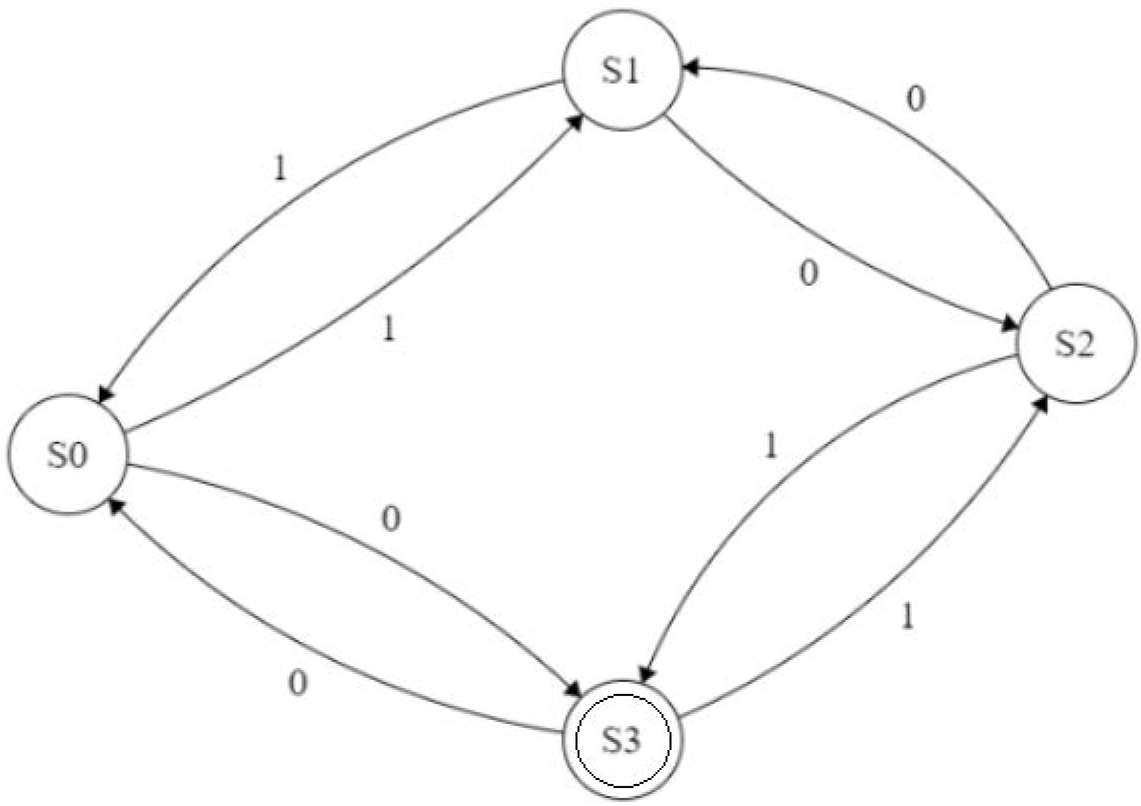


S1 represents the state of seeing the first a
S2 represents the state of seeing ab
And so on…
If we are in S4 (representing abba) and we see an a, the we jump to S1, meaning we saw first a

2) [20 points] Draw a DFA with 4 states that defines the language

$L_2 = \{x \in \{0,1\}^* : x \text{ has even number of 1's and odd number of 0's}\}$

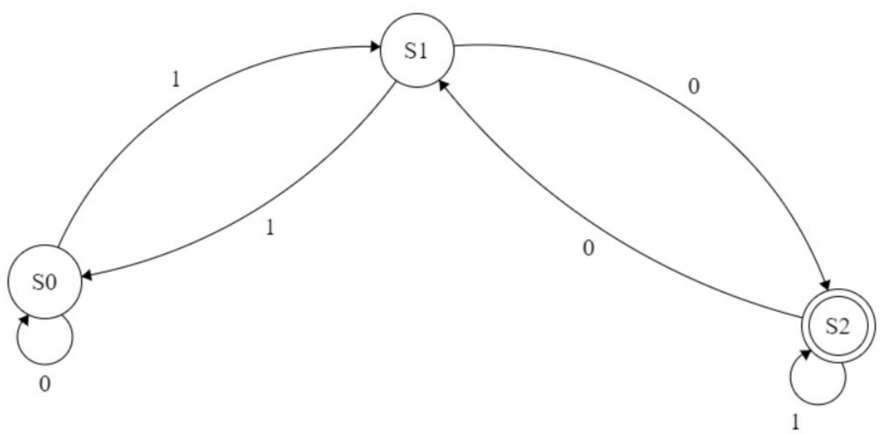
S0 represents the state of even number of 1's and even number of 0's
S1 represents the state of odd number of 1's and even number of 0's
S2 represents the state of odd number of 1's and odd number of 0's
S3 represents the state of even number of 1's and odd number of 0's



3) Recall, a DFA is described using a 5-tuple ($\sum$, S, s_0, δ, F).
Consider the following description of DFA:

$\sum = \{0,1\}$ $S = \{s_0, s_1, s_2\}$ $F = \{s_2\}$	$\delta(s_0,0) = s_0$ $\delta(s_0,1) = s_1$ $\delta(s_1,0) = s_2$ $\delta(s_1,1) = s_0$ $\delta(s_2,0) = s_1$ $\delta(s_2,1) = s_2$

[15 points] Draw the corresponding DFA.
[15 points] ~~Write a regular expression for the language defined by the DFA.~~ Freebie

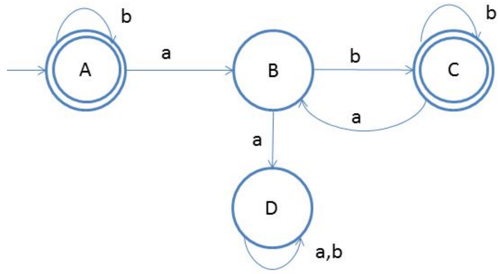


3b) ANSWER: 0* (1 (01*0)* 1 0*)* 1 (01*0)* 01*
Explanation:

1) The basic regexp we can start with is 0*101*. The corresponding transitions in the DFA are: s_0 -> 0* ->s_0 -> 1 -> s_1 -> 0 ->s_2 -> 1* ->s_2
2) As a second step, we note that if we can add (110*) between 0* and the first 1. This corresponds to the "loop" between s_0 and s_1
3) As a next step, we can add (01*0)* inside (110*).
4) Finally, when we are in s_1, we can add (01*0)* before 01*, which corresponds to a "loop" between s_1 and s_2.

For correctness, it is clear that every string accepted by the regexp is also accepted by the DFA.
For the other direction: let x be some string accepted by the DFA. Let's break x into 2 substrings $x = x_1x_2$, , where the partition is based on when the computation of the DFA on x visits s0 for the last time. That is x_2 starts with 1, and is of the form 1 (01*0)* 01*. That is all transitions on x_2 are only between s_1 and s_2. It is not difficult to check that x1 is of the form 0*(1(01*0)*10*)*. Therefore, If x is accepted by the DFA above, then it is of the form:
0* (1 (01*0)* 1 0*)* 1 (01*0)* 01*

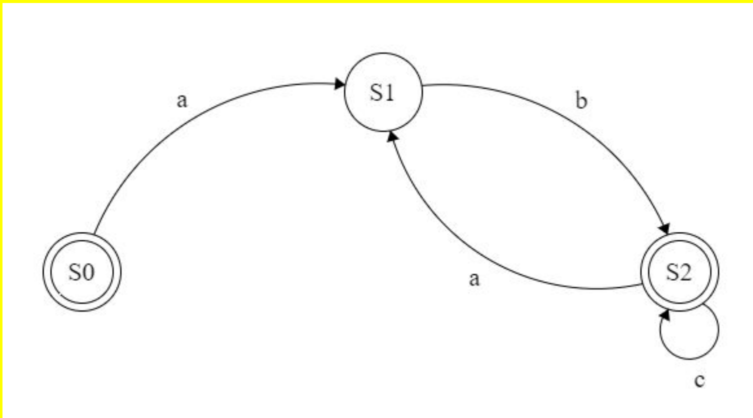
4) [10 points] Describe the language L_4 defined by this DFA.



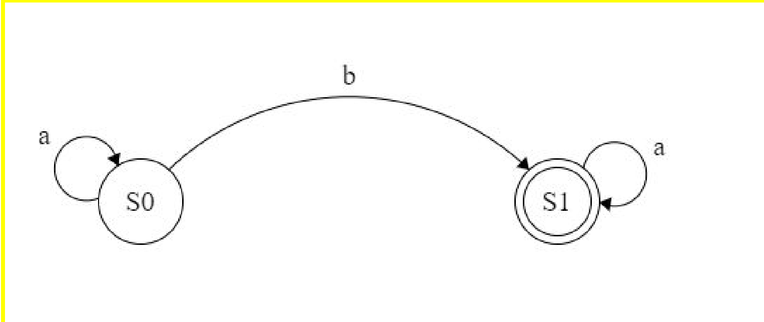
ANSWER:
L = {x ∈:{a,b}* : x does not contain two consecutive a's and ends with b} ∪ {ε}.
For example L contains the words ε, b, bb, bbb, bbbbb… ab, abb, abbb...babb, babab...bbabbab

5) [20 points] For each of the following regular expressions do
- explain in words the language defined by the regular expression
- draw a DFA that defines the language given by the regular expression.

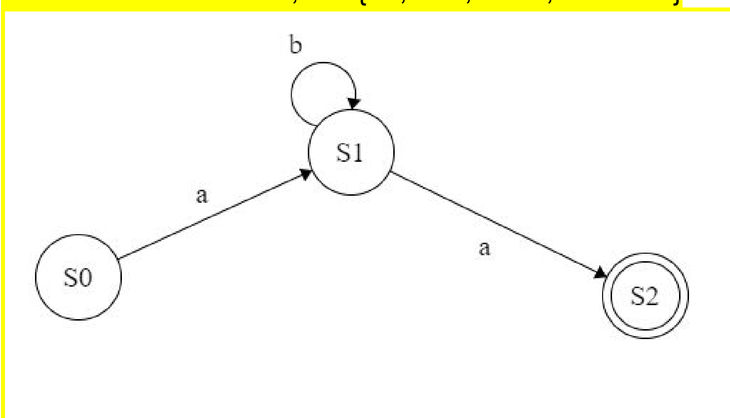
   a. (abc*)*
      ANSWER :
      L = {x : after each a in x comes a b, and there are no two consecutive b's}



b. a*ba*
ANSWER :
L = all strings over the alphabet {a,b} that contains exactly one b



c. ab*a
ANSWER :
L = all strings over the alphabet {a,b} that contain exactly 2 a's, in the beginning and in the end. That is, L = {aa, aba, abba, abbba…}



d. $((1|0)^2)^2$
ANSWER :
L = all binary strings of length 4