CMPT409/815: Advanced Algorithms

Final Exam

Instructor: Igor Shinkar *Decemeber 17, 2020*

- Write your name and SFU ID (numerical and alphabetical) at the top of your solution.
- Submit your solution to Coursys before December 17, 10:00PM.
- Allowed formats for solutions: word, pdf.
- Coursys may give you a warning that you submitted only one file. That's ok.

- **This is an open book exam. You may use books, lecture notes, the internet for hints/solutions**.
- If you find the solution on the internet, you should add the reference. (No penalty for that)
- No matter how you find the solution, you need to explain all your answers.
- Provide enough details to convince me that you understand the solution.

- **You may not go to chegg.com or similar websites and ask for solutions.**
- **You may not discuss your solutions with other students.**

- The exam consists of four (4) problems. The sum of all points in the four problems is 120.
- Explain all your answers. Provide enough details to convince me that you understand the solution.

Question 1 (25 points) *In the* $\mathsf{MAX-RAINBOW-COLORING}_3$ *the input consists of a 3-uniform hypergraph* $G = (V, E)$*, with each hyperedge* $e \in E$ *containing exactly three vertices. Given a coloring of the vertices* $C : V \to \{RED, GREEN, BLUE\}$*, an edge* $e \in E$ *is satisfied if it sees all three colors, that is* $\{C(u) : u \in e\} = \{RED, GREEN, BLUE\}$.

Show a deterministic algorithm that given a 3-uniform hypergraph finds a coloring of the vertices $C : V \to \{RED, GREEN, BLUE\}$ *such that the number of satisfied edges is at least* $2|E|/9$.

Question 2 (25 points) *Given a graph* $G = (V, E)$*, a subset of the vertices* $S \subseteq V$ *is an independent set of* G *if* S *spans no edges.*

(a) Design an algorithm that gets a d*-regular graph* $G = (V, E)$*, and computes a* $\frac{1}{d+1}$*-approximation for the maximum independent set of* G.

(b) Design an algorithm that gets a triangle-free graph $G = (V, E)$ *on* n *vertices and computes a* $\Omega\left(\frac{1}{\sqrt{n}}\right)$*-approximation for the maximum independent set of* G.

(Hint: consider two cases: (1) G *is a vertex of high degree or (2) all vertices have low degree.)*

Question 3 (35 points) *Given a graph $G = (V, E)$, a dominating set of G is a set $S \subseteq V$ such that every $v \in V$ is either in S or is a neighbour of some vertex in S, i.e., $V = S \bigcup (\cup_{s \in S} N(s))$.*

In the weighted minimum dominating set problem the inputs is a graph $G = (V, E)$ and costs of the vertices $(c_v)_{v \in V}$, and the goal is to find a dominating set $S \subseteq V$ such that the total cost of S is minimized, where the total cost of S is $c(S) = \sum_{v \in S} c_v$.

In this question your goal is to design an LP-based approximation algorithm for the weighted minimum dominating set problem. You get full marks if you design an LP-based polynomial time $\ln(n)$-approximation algorithm for the problem. For worse guarantee you will get partial marks.

(a) **[10 points]** *Write an LP relaxation for the weighted minimum dominating set problem.*

(b) **[10 points]** *Write the dual of the LP in item (a).*

(c) **[15 points]** *Show a polynomial time rounding procedure for the LP in item (a), and analyze its approximation guarantee.*

Question 4 (35 points)

(a) **[10 points]** *Let $G = (V, E)$ be a 3-colorable graph. Prove that there exists a bipartition $V = V_1 \cup V_2$ of the vertices of G such that the number of edges crossing the cut is at least $2|E|/3$.*

(b) **[10 points]** *Let $G = (V, E)$ be a 4-colorable graph. Prove that there exists a bipartition $V = V_1 \cup V_2$ of the vertices of G such that the number of edges crossing the cut is at least $2|E|/3$.*

(c) **[15 points]** *Design a polynomial time algorithm that gets a 3-colorable graph $G = (V, E)$ and outputs (with high probability) a bipartition $V = V_1 \cup V_2$ of the vertices of G such that the number of edges crossing the cut is at least $2|E|/3$.*

(Hint: look at the vector-3-coloring of G.)