# CMPT 225 D100, Fall 2025

# Midterm Exam - SOLUTIONS
# October 21, 2025

Name_____

SFU ID: |__|__|__|__|__|__|__|__|__|

| | |
|---|---|
| Problem 1 | |
| Problem 2 | |
| Problem 3 | |
| Problem 4 | |
| TOTAL | |

Instructions:

1. The duration of the exam is 100 minutes.
2. Write your full name and SFU ID **clearly**.
3. This is a closed book exam, no calculators, cell phones, or any other material.
4. The exam consists of four (4) problems, each worth 25 points
5. Write your answers in the provided space.
6. There is an extra page at the end of the exam. You may use it if needed.
7. Explain all your answers.
8. Really, explain all your answers.

Good luck!

**Problem 1 (Java syntax, Big-O notation) [25 points]**

Consider the following class. Assume it has a proper constructor and all setters/getters.

```
class Point {
    int x;
    int y;

    // Constructors,setters,getters

    @Override
    public boolean equals(Object other) {
        if (other == null || getClass() != other.getClass())
            return false;
        Point point = (Point) other;
        return x == point.x && y == point.y;
    }
}
```

A. (5 points) What will be the output of the following code.

```
Point p1 = new Point(1,2);
Point p2 = new Point(1,2);
if (p1 == p2)
   System.out.println("p1 == p2 is true");
if (p1.equals(p2))
   System.out.println("p1.equals(p2) is true");
```

Answer:
if (p1==p2) checks if these are the same objects, hence the answer is false
if (p1.equals(p2)) compares the points using the method .equals(), and will return true.

The output will be only the second line `"p1.equals(p2) is true"`.

B. (5 points) Explain the purpose of the if statement in the code

```
if (other == null || getClass() != other.getClass())
    return false;
```

Answer:
if other == null), no need to check further, because this is not null, so we return false.

This. getClass() returns Point. If other is of a different type than Point, then we do not compare them, and simply return false. (For example, if other is of type Animal, we return false)

C. (5 points) Prove that the running time of the function foo(n) is O(n log(n)).

```
int foo (int n) {
    int sum = 0, i, j;
    for (i=0; i<n; i++)
        for (j=1; j<i; j=j*2)
            sum += 1;
    return sum;
}
```

Answer:
The outer loop has n iterations.
In each iteration of the outer loop, the inner loop makes $\log_2(i) < \log_2(n)$ iterations, each iteration running in O(1) time.
Therefore, the total running time is $n^* \log_2(n)^*O(1) = O(n \log(n))$

D. (10 points) Use big-O notation to express the running time of the function sort() on an array of length n? **Explain your answer.**

```
public static void sort(int array[]) {
    sortHelper(array, 0, array.length-1);
}

public static void sortHelper(int a[], int start, int end) {
    int len = end-start+1;
    if (len<=3) {
        // sort the subarray a[start…end] of length <= 3
        Arrays.sort(a, start, end+1);
        return;
    }
    else { // len>3, we apply recursion
        if  (start < end) {
            int k1 = start + len/3;
            int k2 = start + 2*len/3;
            sortHelper(a, start, k2);
            sortHelper(a, k1, end);
            sortHelper(a, start, k2);
        }
    }
}
```

Answer:
Let T(n) be the running time when the subarray a[strt…end] is of length end-start=n.
We have 3 recursive calls on subarray of length 2n/3.
Hence T(n)=3*T(2n/3) + O(1).

Apply master theorem, we have $c = \log_b(a)=\log_{1.5}(3)$. And the extra term is $O(1)=O(n^0)$.
Since $\log_{1.5}(3)=2.709 > 0$, the total time is $T(n) = n^{2.709}$.
(full marks for the answer $O(n^{\log_{1.5}(3)})$

**Problem 2 (Stack, Queues, Linked Lists) [25 points]**

A. Write a class `QueueOfInts` that supports the operations of a queue, with running time O(1) for each operation.

```
public class QueueOfInts {

   public QueueOfInts() constructor creates an empty queue of ints

   public void enqueue(int item) adds an item to the queue

   public int dequeue() removes an item from the queue and returns it

   public int size()  returns the number of elements in the queue

   public boolean isEmpty() checks if the queue is empty

   public int sum()  returns the sum of all numbers in the queue.
                     The method does not modify the queue.

}
```

Implementing correctly all methods except for `sum()` is worth up to 15 points.

**Before writing code, explain your answer.**

Answer:

We will create a standard queue of ints using the class LinkedList (which implements doubly linked list), and in addition we will hold the variable sum which will hold the current sum of all numbers in the queue.

The private data fields in the class will be

```
private LinkedList<Integer> list;
private int sum;
```

The methods are as follows

```
public QueueOfInts() {
   list = new LinkedList<Integer>();
   sum = 0;
}
```

```
public void enqueue(int item) {
   list.addLast(item);
   sum += item;
}
```

```
public int dequeue() {
   int ret = list.removeFirst();
   sum -= ret;
   return ret;
}
```

```
public int size() {
   // LinkedList stores size as a data field
   return list.size();
}
```

```
public boolean isEmpty() {
   return list.size() == 0;
}
```

```
public int sum() {
   return sum;
}
```

**Problem 3 (Binary Trees) [25 points]**

In this problem use the following definition of Binary Tree. You may assume the classes have the standard getters/setters.

```
public class BTNode<T> {
      private T data;
      private BTNode<T> leftChild;
      private BTNode<T> rightChild;
      private BTNode<T> parent;
   }

public class BinaryTree<T> {
      private BTNode<T> root;
   }
```

A. (10 points) Write the method countLeaves() for the class Binary Tree. The method returns the number of leaves in the tree.
**Explain the idea of the algorithm and its running time**.

```
public int countLeaves() {
     return countLeaves(root);
     // call the recursive function that gets the root
     // and computes the number of leaves under the root
}
```

We will have two helper methods:

```
private boolean isLeaf(BTNode<T> node) {
   return node != null
         && node.getLeftChild()==null
         && node.getRightChild()==null;
}
```

and the recursive function countLeaves:

```
public int countLeaves(BTNode<T> node) {
   if (root == null)
       return 0;
   if (isLeaf(root))
       return 1;
   else
       return countLeaves(root.getLeftChild()) +
                 countLeaves(root.getRightChild());
}
```

B. (10 points) Write the method inOrderTraversal() for the class Binary Tree. The method returns an ArrayList that contains the in order traversal of the tree.
**Explain the idea of the algorithm and its running time**.

```
public ArrayList<T> inOrderTraversal() {
    return inOrderTraversal(root);
    // call the recursive function defined below
}
```

```
public  ArrayList<T> inOrderTraversal(BTNode<T> node) {
   if (node==null)
        return new ArrayList<T>();

   ArrayList<T> ret = inOrderTraversal(node.getLeftChild());
   ret.add(node.getData());
   ret.addAll(inOrderTraversal(node.getRightChild()));
   return ret;
}
```

C. (5 points) Draw ***all*** Binary Trees whose preOrder traversal is [1,2,3,4,5].

Answer: That's a lot of trees to draw. Observe that any binary tree on 5 nodes can be assigned the values 1,2,3,4,5 so that its preOrder traversal is [1,2,3,4,5].

Observe that given a binary tree on 5 nodes, we can run the preOrder traversal on it, and set the values according to that traversal. There are 42 different binary trees with 5 nodes, and each of them can be set the values to get the desired preOrder traversal.

**Problem 4 (Binary Search Trees) [25 points]**

A. (10 points) Consider the class BinarySearchTree, defined as follows.

```
public class BinarySearchTree<T extends Comparable<T>> {
  private BTNode<T> root;
};
```

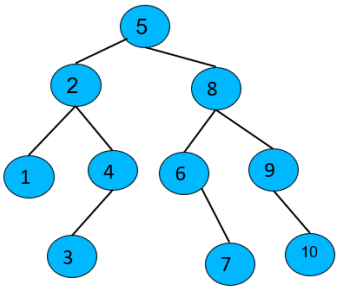
Use the definition of BTNode from Problem 3.

Write the method find() for the class BinarySearchTree. It gets an item, and returns the node containing elt or returns `null` if elt is not in the tree.

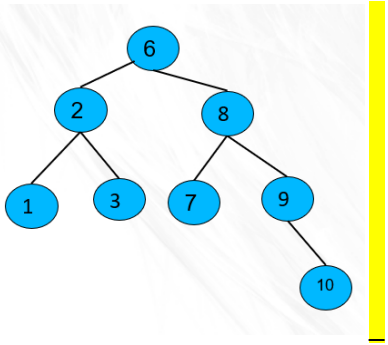
```
public BTNode<T> find(T elt) {

    BTNode<T> current = root;
    while (current != null) {
       if (current.getData().equals(elt))
           return current;
       else if (current.getData().compareTo(elt) > 0)
           current = current.getLeftChild();
       else // (current.getData().compareTo(elt) < 0)
           current = current.getRightChild();
    }
    return null;

}
```

B. (5 points) Given the Binary Search Tree below.

- Remove 4 and 5 from the tree and draw the result.




Answer:

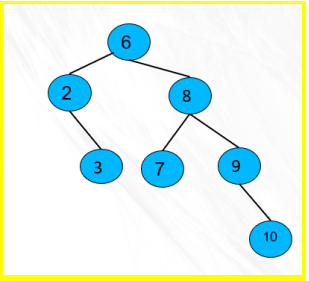



Removing 4: we simply pull 3 up
Removing 5: we copy 6 to the root, and then remove the node 6, by connecting 7 to 8

C. (10 points each question) For each of the following statements, decide if it is True or False. **Prove your answer**.

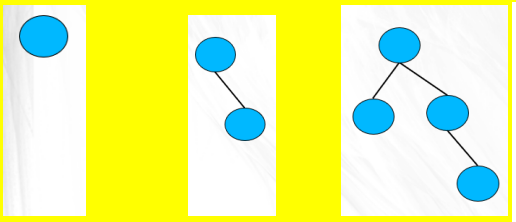
1. There exists an AVL tree of height 3 (i.e., with 4 levels) with at most 3 leaves.

Answer:




2. There exists an AVL tree of height 6 with 13 leaves.

Answer: Yes. Consider a minimal AVL tree of height h, and let L(h) be the number of leaves in it. h=0 h=1 h=2 …



For h=0 we have a tree with L(0)=1 leaf
For h=1 we have a tree with L(1)=1 leaf
For h=2 we have a tree with L(0)+L(1)=2 leaves
For h=3 we have a tree with L(1)+L(2)=3 leaves
For h=4 we have a tree with L(2)+L(3)=5 leaves
For h=5 we have a tree with L(3)+L(4)=3+5=8 leaves
**For h=6 we have an AVL tree with L(4)+L(5)=5+8=13 leaves**

**Extra page**

**Master Theorem**

Let $T(n) = a\,T(n/b) + f(n)$, $T(1) = O(1)$

Define $c = \log_b(a)$

- If $f(n) = O(n^d)$ for $d<c$, then $T(n) = \Theta(n^c)$
- If $f(n) = \Omega(n^d)$ for $d>c$, then $T(n) = \Theta(n^d)$
- If $f(n) = \Theta(n^c)$, then $T(n) = \Theta(n^c \log(n))$