CMPT881: Special Topics in Theoretical Computer Science

Homework assignment 1

Instructor: Igor Shinkar — *Due date: February 7, 2019*

Question 1 *Denote by $\alpha(G)$ the size of the maximum independent set in G.*

Design an algorithm that given a graph G that contains an independent set of size $\alpha(G) \geq (\frac{1}{2}+\delta)|V|$, outputs an independent set of size $2\delta|V|$.

Question 2 *Design a $\left(2 - \frac{\log(n)}{k}\right)$-approximation algorithm for finding a k-vertex cover in a given graph whose running time is polynomial in the size of the input.*

Question 3 *A t-uniform hypergraph $H = (V, E)$ is a collection of vertices V, and a collection of hyperedges E, where each hyperedge $e \in E$ is a subset of V of size $|e| = t$. The case of $t = 2$ corresponds to graphs.*

Consider the Minimum Vertex Cover *problem for t-uniform hypergraphs: Given a t-uniform hypergraph the goal is to find a collection of vertices $C \subseteq V$ such that for every $e \in E$ it holds that $e \cap C \neq \emptyset$. Design a t-approximation algorithm for the problem of finding a minimum vertex cover in a hypergraph.*

Question 4 *Design a polynomial time algorithm that gets a satisfiable 3-CNF formula with n variables and m clauses, and finds an assignment that satisfies at least $\frac{7m}{8} + \log(m)$ clauses.*

You may assume that there exists a poly-time algorithm that when given a CNF formula, where each clauses consists of at most 3 variables, finds an assignment that satisfies at least 7/8-fraction of the clauses.

Question 5 *Recall the 2-approximation algorithm for metric TSP we saw in class. For any graph G denote by $costALG(G)$ the cost of the solution that the algorithm outputs and denote by $optTSP(G)$ the cost of the optimal TSP solution on G Show that for any constant $c < 2$ there is a graph G such that $costALG(G) \geq c \cdot optTPS(G)$.*

Question 6 *Recall the Christofides' algorithm that gives a 1.5-approximation algorithm for metric TSP we saw in class. For any graph G denote by $costCHR(G)$ the cost of the solution that the algorithm outputs and denote by $optTSP(G)$ the cost of the optimal TSP solution on G Show that for any constant $c < 1.5$ there is a graph G such that $ccostCHR(G) \geq c \cdot optTPS(G)$.*