CMPT 706, Spring 2020
Quiz 1 - January 30

Name_________________________

SFU ID: |__|__|__|__|__|__|__|__|__|

Instructions:

1. Write your name and SFU ID **clearly**.
2. No calculators, no cell phones, or any other material.
3. Write your answers in the provided space.
4. Explain all your answers.

Question 1 (10 points)

Let $f, g : N \to N$ be two functions on positive integers that output positive integers. Suppose that $f = O(h)$ and $g = \Omega(h)$. Is it true that $f = O(g)$?
If true, prove it. If false, provide a counterexample.

This is true.

Proof:

By definition $f = O(h)$ means that there is some constant $C_1 > 0$ and $k_1 > 0$ such that for all $n > k_1$ we have $f(n) \leq C_1 \cdot h(n)$.

By definition $g = \Omega(h)$ means that there is some constant $C_2 > 0$ and $k_2 > 0$ such that for all $n > k_2$ we have $g(n) \geq C_2 \cdot h(n)$.

Therefore, by letting $k = max(k_1, k_2)$ and $C = C_1 \cdot C_2$ we get that for all $n > k$ we have $f(n) \leq C_1 \cdot h(n) \leq C_1 \cdot C_2 \cdot h(n) = C \cdot h(n)$, and hence $f = O(g)$.

Question 2 (10 points)

Show the execution of the Euclidean Algorithm for computing gcd(144,54).
Write explicitly all intermediate steps of the algorithm.

gcd(144, 54) = gcd(144 mod 54, 54) = gcd(36, 54) = gcd(54, 36) = gcd(54 mod 36, 36)
= gcd(18, 36) = 18

Question 3 (10 points)

Compute the product 43·71 using the fast multiplication algorithm that performs only 3 multiplication on decimal digits.

U = 3*1 = 3
H = 4*7 = 28
T = (4+3)*(7+1) - U - H = 7 *8 - 3 - 28 = 56-3-28 = 25

Therefore 43*71 = 28*100 + 25*10 + 3 = 3053