CMPT 706 — Algorithms for Big Data

Homework Assignment 3

Instructor: Igor Shinkar *Due date: Tuesday, March 3, 2020*

Submit your solutions, printed or written in readable handwriting, to the assignment boxes in CSIL.

Question 1 (20 points) *Design an algorithm that gets as input (1) a 2×2 matrix A with entries in $[0, \ldots, 99]$, and (2) a positive integer n. The algorithm computes $A^n = \underbrace{A \cdot A \cdot \ldots \cdot A}_{n \text{ times}}$ such that each entry in the answer is modulo 100. The runtime of the algorithm must be $O(log(n))$.*

What is the runtime of the algorithm in terms of the entire length of the input?

Question 2 (20 points) *Consider the following matrix.*

$$\begin{bmatrix} & A & B & C & D & E & F & G & H \\ A & 0 & 1 & 0 & 0 & 0 & 1 & 0 & 0 \\ B & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 \\ C & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ D & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 \\ E & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ F & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ G & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ H & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(a) Draw the directed graph described by this matrix.

(b) Perform depth-first search on each of the following graphs. Whenever there's a choice of vertices, pick the one that is alphabetically first.

(c) Classify each edge as a tree edge, forward edge, back edge, or cross edge.

(d) Write the pre and post number of each vertex.

Question 3 (20 points)

(a) Consider an execution of the DFS algorithm on K_n, the complete graph on n vertices. What will be the shape of the DFS tree? Does the shape depend on the order in which the vertices are explored?

(b) Consider an execution of the DFS algorithm on C_n, the cycle graph on n vertices. What will be the shape of the DFS tree? Does the shape depend on the order in which the vertices are explored?

(c) Consider an execution of the BFS algorithm on K_n, the complete graph on n vertices. What will be the shape of the BFS tree? Does the shape depend on the order in which the vertices are explored?

(d) Consider an execution of the BFS algorithm on C_n the cycle graph on n vertices. What will be the shape of the BFS tree? Does the shape depend on the order in which the vertices are explored?

(e) Is the following statement true? For an undirected graph $G = (V, E)$ and an edge $(u, v) \in E$, if during execution of the DFS algorithm we get $post(u) < post(v)$, then v is an ancestor of u in the DFS tree. If true, prove it. If not, give a counterexample

Question 4 (20 points) *Design a linear-time algorithm, that gets as input an undirected graph $G = (V, E)$ and a particular edge $e \in E$, and determines whether G has a cycle containing the edge e. The running time of your algorithm on a graph $G = (V, E)$ should be $O(|V| + |E|)$.*

Question 5 (20 points) *Modify the pre_visit() and post_visit() functions in the DFS algorithm we saw in class so that for each vertex the algorithm prints its distance from the root in the DFS tree. The runtime of the algorithm on any graph $G = (V, E)$ must be $O(|V| + |E|)$. Explain exactly which variables you need to add, and how to modify the pre_visit() and post_visit() functions.*