CMPT 706 — Algorithms for Big Data

Homework Assignment 5

Instructor: Igor Shinkar *Due date: Tuesday, April 7, 2020*

Submit your solutions to Coursys in pdf format.

Question 1 (20 points) *Consider the following algorithm BAR that gets a positive integer and works as follows:*

BAR(int n):
- if (n <=0), return 0;
- else, return (n - BAR(BAR(n-1)));

[5 points] What will be the output of BAR(3)?

[15 points] Write an algorithm that has the same functionality as BAR, so that the runtime of $BAR(n)$ is $O(n)$?

Question 2 (20 points) *Run the dynamic programming based algorithm for the (unweighted) edit distance problem we saw in class on inputs $X = EXCELLENCE$ and $Y = SILENCE$. Draw the matrix representing the computation.*

Question 3 (20 points) *Write an algorithm for the weighted version of the edit distance problem. That is, the input is an alphabet Σ, replacement cost $w_{\sigma,\tau}$ for every pair of different letter $\sigma, \tau \in \Sigma$, and insertion/deletion cost z_σ for every letter $\sigma \in \Sigma$. The algorithm, then, gets two strings X, Y over the alphabet Σ and finds the weighted edit distance between X and Y. The runtime of the algorithm must be $O(|X| \cdot |Y|)$.*

Question 4 (20 points) *Write a dynamic programming algorithm that gets two strings X and Y, and finds their longest common (consecutive) substring. For example, if the strings are $X = CloseTheDoorPlease$ and $Y = TheDoorIsOpen$, then the output should be $TheDoor$. The algorithm must run in time $O(|X| \cdot |Y|)$.*

Question 5 (20 points) *Consider the following change making problem: The input is an infinite supply of coins of m given values, and a target n. You goal is to find the minimal number of coins whose values sum up to n. For example, if the given 4 types of coins with values: $2, 3, 5, 7$ and the target is 13, then this target can be achieved using 3 coins: 3+5+5 or 7+3+3.*

Write a dynamic programming algorithm for solving this problem. What is the runtime of your algorithm?