CMPT404/705: Design and Analysis of Computing Algorithms

Homework Assignment 2

Instructor: Igor Shinkar *Due date: March 04, 2022*

Question 1 (3 points) *For each of the recursive relations compute its order of growth.*

(a) Let $T_a(n) = 6T_a(n/2) + O(n^2)$, $T_a(1) = O(1)$.

(b) Let $T_b(n) = 2T_b(2n/5) + T_b(n/5) + O(n)$, $T_b(1) = O(1)$.

(c) Let $T_c(n) = 2T_c(n/2) + O(n\log(n))$, $T_c(1) = O(1)$.

Question 2 (3 points) *Recall the Karatsuba algorithm for multiplying two integers of length n in time $O(n^{\log_2(3)})$. Let's say the algorithm works as follows: if $n \geq 2$, the algorithm makes recursive calls, and if $n = 1$, the algorithms just computes the product of the two input digits and returns the answer.*

Show the execution of the algorithm on the input 1483×5293. Write down explicitly all the recursive calls made by the algorithms.

Question 3 (3 points) *Consider the problem of counting of inversions in an array. The input is an array $[a_1, a_2, \ldots, a_n]$ of distinct real numbers. For indices $i < j$ we say that the pair (i, j) is an inversion if $a_i > a_j$.*

Design an algorithm that gets an array of n distinct numbers and returns the number of inversions in time $O(n\log(n))$.

(Hint: use divide and conquer approach by splitting the array into two halves, and solving recursively the problems in the left half and in right half. How do you count the number of inversions (i, j) with $i < n/2$ and $j > n/2$?)

Question 4 (3 points) *Recall the deterministic algorithm for finding the k'th smallest element we saw in class. The algorithm finds a pivot by partition an array into 5-tuples, and computing the median of the medians. Then it uses the pivot to recursively compute the k'th smallest element.*

(a) *Consider the variant where instead of looking at 5-tuples, we partition our array into 7-tuples, and take the median of the medians. What will be the running time of the algorithm with this variant? Write the recursive expression for the running time, and solve it.*

(b) *Consider the variant where instead of looking at 5-tuples, we partition our array into 3-tuples, and take the median of the medians. What will be the running time of the algorithm with this variant? Write the recursive expression for the running time, and solve it.*

Question 5 (3 points) *The input is consists of two sets A and B of integers with values between 0 and n. For each $0 \leq k \leq 2n$ let $N_k = |\{(a,b) : a \in A, b \in B, a + b = k\}|$. Design an algorithm that given the sets A and B, finds the numbers $(N_k)_{k=0}^{2n}$ in time $O(n\log(n))$*