Analysis of Sparse-Partial Wavelength Conversion in Wavelength-Routed WDM Networks

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Abstract — **Wavelength conversion has been shown as one of the key techniques to improve blocking performance in a wavelength-routed all-optical network. Given that wavelength converters nowadays remain very expensive, how to make effective use of a limited number of wavelength converters becomes an important issue. In this paper, we propose Sparse-Partial Wavelength Conversion (SPWC) network architecture with the inherent flexibility that can facilitate network carriers to migrate the optical backbone to support wavelength conversion. We demonstrate that this network architecture can significantly save the number of wavelength converters, yet achieving excellent blocking performance. Theoretical and simulation results indicate that, the performance of a wavelength-routed WDM network with only 1-5% of wavelength conversion capability is very close to that with Full-Complete Wavelength Conversion capability.**

I. Introduction

Wavelength-routed all-optical WDM networks are considered to be candidates for the next generation wide-area backbone networks [3]. A physical wavelength-routed network consists of a set of wavelength routers connected by fiber links. Each fiber link can support a number of wavelength channels by using Wavelength Division Multiplexing (WDM); and the wavelength routers can switch the input optical signals according to their wavelengths. Two wavelength routers, whether they are physically adjacent or not, can communicate with each other by setting up a "lightpath" in between, which is a direct optical connection without any intermediate electronics. In a dynamic wavelength-routed WDM network, a sequence of lightpath requests arrives over time and each lightpath has a random holding time. Due to the capacity limitation of the network, some lightpath requests may not be satisfied, resulting in blocking. One of the primary design objectives in dynamic wavelength-routed all-optical networks is thus to minimize this blocking probability.

Establishing a lightpath generally requires the same wavelengths to be allocated on all the fiber links along the path. This is known as the *wavelength continuity constraint*, which makes the modeling of wavelength-routed networks different from that of traditional circuit-switched telephone networks. Such constraint can be eliminated by using wavelength converters, which can convert the optical signal from one wavelength to another [7]. A wavelength router with conversion capability is called a *wavelength-convertible router*, or WCR. However, wavelength converters remain very expensive nowadays; hence, different architectures of WCRs have been proposed to save the cost:

Complete Wavelength Conversion: Fig. 1 shows an example of a WCR with Complete Wavelength Conversion capability, where each output port of the optical switch is associated with a dedicated wavelength converter. This kind of ideal WCR is assumed to be able to convert all the input wavelengths to any other wavelengths simultaneously without any limitation. Note that the number of converters is equal to the number of the fiber links multiplied by the number of wavelengths per fiber. Since the number of wavelengths on each fiber could be hundreds or even more, the number of converters inside a WCR will be very large and the cost of such architecture can be prohibitively high.

Partial Wavelength Conversion: It has been shown that a WCR with a limited number of converters can achieve very close performance to Complete Wavelength Conversion. This is referred to as Partial Wavelength Conversion [1,8]. Fig. 2 shows the architecture of a WCR with share-per-node partial wavelength conversion [8]. There is a pool of wavelength converters which are shared by all the output ports. This architecture requires much less number of wavelength converters. However, it is more complex than a wavelength router without wavelength conversion, because it needs an addition small optical switch (OSW). In addition, it remains unknown how many converters should be equipped in a WCR in order to achieve satisfactory performance.

If all the wavelength routers in the network support wavelength conversion (either complete conversion or partial conversion), we call it *Full Wavelength Conversion*. On the other hand, if only a small part of the wavelength routers can perform wavelength conversion, the network is called with S*parse Wavelength Conversion* [10]. The latter has received much attention recently, because it can significantly save the number of WCRs. It also offers a flexible solution for the network carriers to upgrade their network gradually to support wavelength conversion. To date, most of existing studies simply assume that, the WCRs in a Sparse Wavelength Conversion networks all have the capability of Complete Wavelength Conversion, which however is very costly and inefficient in practice.

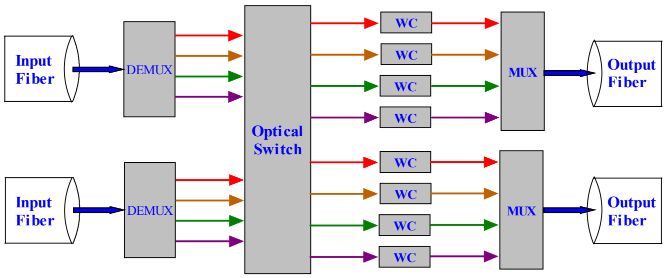


Fig.1. A wavelength converter with full wavelength conversion

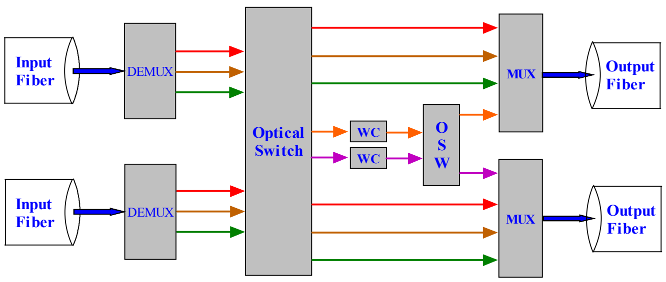


Fig. 2. A wavelength converter with partial wavelength conversion

In this paper, we first analyze Partial Wavelength Conversion that leads to the following observations: First, in order to achieve small blocking probability, over-provisioning is usually done in the backbone network. This implies that only a relatively small portion of the overall network capacity is used to carry the traffic. Second, only the lightpaths that pass through a WCR could require allocating a wavelength converter on this WCR. Hence, as long as the number of the bypassing lightpaths is not large, a small number of converters are enough. Third, a wavelength assignment algorithm, if carefully designed, can further save the number of converters, and most of the lightpaths can be setup successfully without wavelength conversion.

Based on such observations, we propose the *Sparse-Partial Wavelength Conversion* (SPWC) network architecture, which aims to combine the advantages of Partial Wavelength Conversion and Sparse Wavelength Conversion. In such networks, only part of wavelength routers are WCRs with Partial Wavelength Conversion, while other wavelength routers have no wavelength conversion capability. This architecture has two important advantages: 1) it can significantly reduce the number of wavelength converters needed; 2) it is very flexible for the network carrier to migrate their network to support wavelength conversion, either by adding more converters into the WCRs, or by replacing the old wavelength routers with new WCRs.

Although wavelength converter placement problem has been extensively studied for the Sparse Wavelength Conversion case [4,5,11-13], the corresponding problem for the SPWC case is quite different. To this end, we re-define the problem for SPWC network architecture and propose a simple but effective scheme to solve it. Theoretical and simulation results demonstrate that only 1-5% number of wavelength converters, if appropriately placed, is needed to achieve comparable performance to that of Full-Complete Wavelength Conversion.

The rest of the paper is organized as follows. In Section II, we present quantitative analysis on why Partial Wavelength Conversion can usually achieve almost the same performance as Complete Wavelength Conversion. In Section III, we describe the proposed SPWC network architecture, and then investigate the wavelength converter placement problem. Numerical results are presented in Section IV. Finally, Section V concludes the paper.

II. ANALYSIS OF PARTIAL WAVELENGTH CONVERSION

In this section, we analyze the Partial Wavelength Conversion and show why it can achieve very good blocking performance compared to Complete Wavelength Conversion. Our key observations in this section are: (1) under a small blocking probability, the total network traffic carried in the network has to remain relatively low. Hence, the number of lightpaths concurrently passing through a wavelength router is relatively small as compared to its theoretical capacity. (2) a well-designed wavelength assignment algorithm can further decrease the number of wavelength converters.

The above two observations serve as the basis for our SPWC architecture.

A. Network Assumptions and notations

We first give some assumptions and notations for our network model, as follows:

1. An arbitrary mesh WDM network consists of N nodes and J fiber links. The nodes are labelled from 1 to N, and the links are labelled from 1 to J.
2. The nodal degree of node n is denoted by $D(n), 1 \le n \le N$.
3. The number of converters inside node n is denoted by $F(n)$.
4. For simplicity, we consider bi-directional links. Each link can support W wavelengths in both directions.
5. We assume that lightpath connection requests for end-to-end node pair a follows a Poisson process with rate A_a. We also assume that the connection holding times are exponentially distributed with a unit time. The total traffic offered to the network is T Erlangs.

6. For simplicity, we assume the fixed shortest path routing algorithm is used. The route between node pair a is denoted by R_a, and the length of the route in hop-count is $h(R_a)$. We further define that the ith link of route R_a is $R_a(i), 1 \le i \le h(R_a)$.
7. The blocking probability of route R_a is denoted by B_{R_a}.

B. Calculation of Overall Blocking Probability

We now show a simple model to calculate the blocking probability of a wavelength-routed WDM network with Full-Complete Wavelength Conversion. In such a network, each node is a WCR with Complete Wavelength Conversion, i.e., $F(n) = D(n)W$, and there is no wavelength continuity constraint.

The overall blocking probability B is defined as the ratio of the blocked traffic to offered traffic. That is,

$$B = \frac{\sum_a A_a B_{R_a}}{\sum_a A_a}. \quad (1)$$

To obtain the steady-state probability of the number of available wavelengths on each link, we use the reduced-load approximation method presented in [2]. Let X_j denote the random variable representing for the number of free wavelengths on link j. We assume that random variables $X_j, j \in \{1, \ldots, J\}$ are independent, and the call requests arriving at link j follow a Poisson process with rate α_j. Let $q_j(m_j)$ denote the probability that m_j wavelengths are free on link j. According to our assumptions, the arriving and serving behavior on the link forms an $M/M/m/m$ (m-server loss) system and the corresponding Markov chain is illustrated in Fig. 3. Solving the Markov chain, we have

$$q_j(m_j) = P(X_j = m_j) = \frac{\prod_{i=1}^{m_j}(W - i + 1)}{\alpha_j^{m_j}} P(X_j = 0). \quad (2)$$

and

$$q_j(0) = P(X_j = 0) = \left[1 + \sum_{m_j=1}^{W} \frac{\prod_{i=1}^{m_j}(W - i + 1)}{\alpha_j^{m_j}}\right]^{-1}. \quad (3)$$

Following the approximation made in [6] for the carried traffic on link j, we can determine α_j using the following equation,

$$\alpha_j(1 - q_j(0)) = \sum_{\substack{a, \text{ where link } j \\ \text{belongs to } R_a}} A_a(1 - B_{R_a}). \quad (4)$$

A lightpath can be setup on a route if and only if every link on the route has free wavelengths. Thus we can calculate the blocking probability of a route according to the following equation:

$$B_{R_a} = 1 - \prod_{i=1}^{h(R_a)} (1 - q_{R_a(i)}(0)). \quad (5)$$

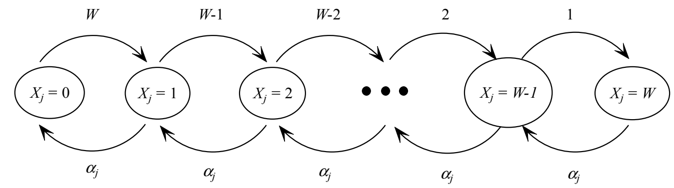


Fig. 3. Markov chain for the distribution of free wavelengths on link j

The above equations lead to a set of fixed-point non-linear equations, which can be solved by iterative substitutions, as follows:

1. Initialize B_{R_a} to 0 for all routes, and $q_j(0)$ to 0 for all links.
2. Determine α_j using Eq. (4) for all links.
3. Determine $q_j(m_j)$ using Eqs. (2) and (3) for all links.
4. Determine B_{R_a} for all routes using Eq. (5). If the new values of B_{R_a} converge to the old values, the iteration is terminated and we can go to Step (5). Otherwise go to Step (2) for the next iteration.
5. Finally, determine the overall blocking probability B using Eq. (1).

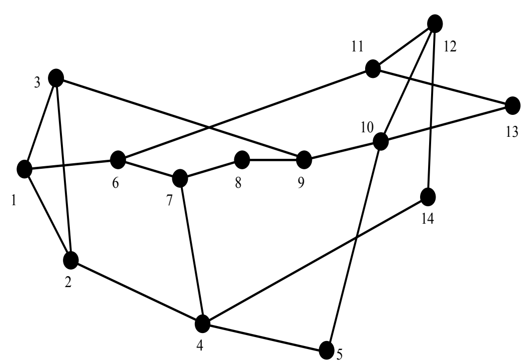


Fig. 4. 14-node NSFNET network

For illustration, we use the above method to calculate the blocking probability for the 14-node NSFNET (Fig. 4) topology. We assume that the traffic load is uniformly distributed to all the node pairs. Table 1 shows the total traffic which can be carried for different number of wavelengths, given a blocking probability of 2%. Furthermore, suppose the average route length is denoted by L, we can estimate the average wavelength utilization U using $U = \frac{T*(1-B)*L}{W*J}$. According to our shortest path routing scheme, the average route length L is 2.18. We can observe that the average wavelength utilization is only around 60%.

Table 1 Total traffic that can be carried on NSFNET when $B = 2\%$

Number of Wavelengths: W	40	50	60	70	80	90	100
Total Traffic in Erlangs: T	208	270	333	397	460	525	590
Wavelength Utilization: U	56%	58%	59%	61%	61%	62%	63%

C. Analysis of Node Bypassing Traffic

After knowing how much traffic the network can handle, we can further analyze the traffic bypassing each node. A lightpath does not need wavelength conversion at its two end nodes; thus for each node, only the bypassing lightpaths could potentially need wavelength conversion. Hence, what we are interested is the number of concurrent lightpaths bypassing each node, which does not count the lightpaths that are generated by or terminated at that node. We assume that the bypassing lightpaths arriving at node n follow a Poisson process with rate β_n. It is straightforward that

$$\beta_n = \sum_{\substack{a,\text{ where route } R_a \\ \text{bypasses node } n}} A_a \quad . \tag{6}$$

Let $p_n(f_n)$ denote the probability that f_n lightpaths are concurrently bypassing node n. So f_n varies from 0 to $F(n)$ because node n can support at most $F(n)$ concurrent lightpaths at any time. According to the assumptions, the arrival and departure behavior of the bypassing lightpaths on each node also forms an $M/M/m/m$ system. By solving the Markov chain, we have

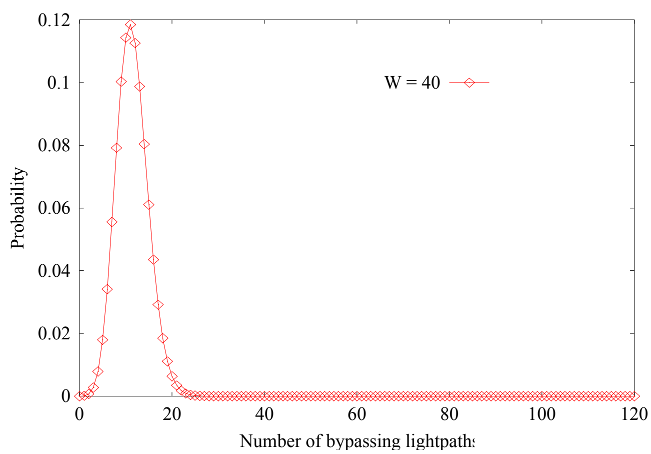
$$p_n(f_n) = \frac{(\beta_n)^{f_n}\dfrac{1}{f_n!}}{\sum_{i=0}^{F(n)}(\beta_n)^i\dfrac{1}{i!}}, \quad 0 \le f_n \le F(n) . \tag{7}$$

Given that the blocking probability of the $M/M/m/m$ system is very low, the $M/M/m/m$ system can be approximated by an $M/M/\infty$ system. Hence, the average number of bypassing lightpaths can be approximated by β_n, because the service time is assumed to be exponentially distributed with unit time. We still use NSFNET as an example. Assume that 40 wavelengths are available, and the blocking probability requirement is 2% or less. From the results of Section II-B, the total traffic is 208 Erlangs and each node pair has 2.286 Erlangs of lightpath requests. We can therefore calculate the bypassing traffic for each node using Eq. (6). The results are shown in Table 2.

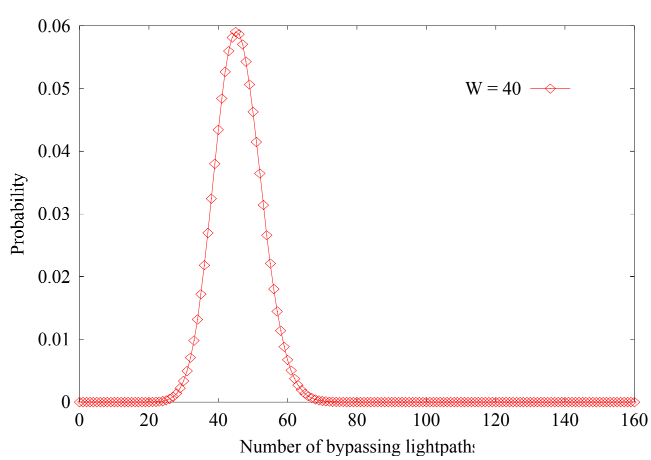
We find that the ratio of $\dfrac{\beta_n}{F(n)}$ ranges from 0 to 28.6%, and in most cases it is only about 10-15%. This implies that the number of lightpaths concurrently bypassing a node is very small compared to the node's capacity. We also show the curves of $p_n(f_n)$ in Fig. 5 (a) and (b), for nodes 1 and 4 respectively, which give the probability distribution of the number of bypassing lightpaths. We can see that the probability that more than 20 lightpaths are concurrently bypassing node 1 is almost zero. Therefore, in node 1, 20 wavelength converters are enough to achieve the same performance as Complete Wavelength Conversion which requires 120 converters. For node 4, which has the highest volume of bypassing traffic, the probability that more than 70 lightpaths are concurrently bypassing it is almost zero.

Table 2 β_n and $F(n)$ of NSFNET when total traffic = 208 Erlangs

Node n	*1*	*2*	*3*	*4*	*5*	*6*	*7*
β_n	11.4	18.3	11.4	45.7	11.4	27.4	25.1
$F(n)$	120	120	120	160	80	120	120
Node n	*8*	*9*	*10*	*11*	*12*	*13*	*14*
β_n	2.3	18.3	36.6	16.0	18.3	0	4.6
$F(n)$	80	120	160	120	120	80	80



(a) Node 1, $F(n)$ =120



(b) Node 4, $F(n)$ =160

Fig. 5. Probability distribution of the number of bypassing lightpaths

To conclude, from a node's perspective, considerable percentage of lightpaths is not bypassing. Since only the bypassing lightpaths may require wavelength conversion, it is possible to equip a small number of wavelength converters in each node to achieve satisfactory performance.

D. Wavelength Assignment

In the previous subsection, we conclude that only a small number of lightpaths bypass a node concurrently. In this subsection, we further show that most of these bypassing lightpaths do not need wavelength conversion if an appropriate wavelength assignment is employed.

We conduct simulations for the NSFNET topology without wavelength conversion, *i.e.*, each lightpath has to use the same wavelength on all its links. The total network traffic is 208 Erlangs and 1,000,000 lightpath requests are generated. We use the First-fit wavelength assignment scheme in our simulation [14]. For each node, we get the percentage of the bypassing lightpaths that are set up successfully, as shown in Table 3. We observe that more than 90% of the bypassing lightpaths can be set up without wavelength conversion by using the simple First-fit wavelength assignment scheme. In other words, no more than 10% of the bypassing lightpaths actually need wavelength conversion. Recall the results of Section II-C that the number of concurrently bypassing lightpaths on node n are much less than the value of $D(n)W$, we can therefore conclude that, a very small number of wavelength converters can achieve almost the same performance as complete wavelength conversion.

Table 3 The percentage of the bypassing lightpaths that are setup successfully without wavelength conversion

Node *n*	*1*	*2*	*3*	*4*	*5*	*6*	*7*
Per.	96.9%	95.3%	97.8%	93.9%	96.0%	92.9%	92.3%
Node *n*	*8*	*9*	*10*	*11*	*12*	*13*	*14*
Per.	100%	95.0%	95.5%	94.8%	96.0%	100%	97.5%

III. SPARSE-PARTIAL WAVELENGTH CONVERSION

In this section, we propose the SPWC network architecture based on the observations made in the previous section.

A. Sparse-Partial Wavelength Conversion

Given that wavelength conversion technology remains immature, it is not practical for the network carrier to replace all the wavelength routers by WCRs. Motivated by the literature results that Sparse Wavelength Conversion can achieve very good performance compared to Full Wavelength Conversion, we propose the SPWC network architecture which combines the advantages of Partial Wavelength Conversion and Sparse Wavelength Conversion. There are two kinds of nodes in such network: nodes without wavelength conversion capability, and nodes with Partial Wavelength Conversion capability. By using Sparse Wavelength Conversion and Partial Wavelength Conversion together, only a small number of wavelength converters are needed to achieve comparable performance as Full-Complete Wavelength Conversion. As such, it only requires that a small fraction of wavelength routers be replaced with WCRs, which is very flexible for the network carriers to migrate the existing network to support wavelength conversion.

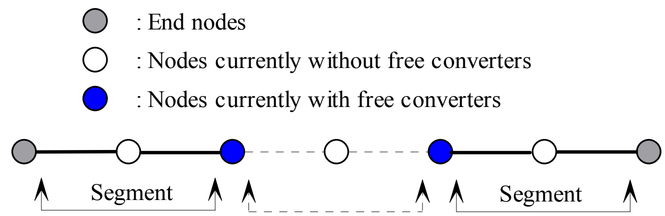


Fig. 6. A lightpath and its segments

A SPWC network operates as follows: Upon the arrival of a lightpath request, if there is any link in the selected route which currently has no free wavelength, we can not set up the lightpath on this route. Otherwise, we should first try to find a common free wavelength on all the links along the selected path. We have shown in Section II-D that, most of the lightpath requests can be setup in this way without using any wavelength converters. If there is no common free wavelength, we then check whether wavelength converters can help. A lightpath is divided into several segments by the intermediate WCRs which currently have free converters, as shown in Fig. 6. Notice that, a WCR will have no conversion capability if all its wavelength converters have been allocated. Notice that, each segment still suffers the wavelength continuity constraint. A lightpath can be setup successfully if and only if every segment has common free wavelength(s). Hence we have to check whether there exist common free wavelengths for each segment. Wavelength converters will then be allocated if necessary. Once the lightpath is terminated, the allocated converters will be released.

B. Analytical Model

In this subsection, we propose an approximate analytical model to derive the blocking probability of a wavelength-routed WDM network with the SPWC architecture.

Suppose the number of WCRs in route R_a is *D*, excluding the two end nodes of R_a. Since the WCR has only the partial conversion capability, each WCR has two states: (1) with no free wavelength converter; (2) with one or more free wavelength converters. Hence, there are altogether 2^D different conversion states for route R_a. For each conversion state *X*, we denote the number of WCRs with free wavelength converters by E_X, where $0 \le E_X \le D$. Therefore, for conversion state *X*, route R_a is divided into $E_X + 1$ segments, represented by $S_0, S_1, \ldots, S_{E_X}$, respectively. Each segment should use the same wavelength on its links. We introduce $u_{S_k}(i)$ for representing the probability that i wavelengths are common free on segment S_k. A lightpath will be setup on the route successfully if and only if each segment has at least one common free wavelength. Let $B_{R_a,X}$ denote the route blocking probability for conversion state *X*, we can have:

$$B_{R_a,X} = 1 - \prod_{k=0}^{E_X} \left[1 - u_{S_k}(0)\right]. \qquad (8)$$

The route blocking probability is thus given by

$$B_{R_a} = \sum_{X} \left\{ B_{R_a,X} P(X) \right\}, \qquad (9)$$

where *P(X)* is the probability of conversion state *X*. Therefore, in order to calculate B_{R_a}, we have to calculate $u_{S_k}(i)$ and *P(X)*.

Let us first show how to calculate $u_{S_k}(i)$, which is the probability that i wavelengths are common free on segment S_k.

If S_k is a two-hop segment composed by link j_1 and j_2. Then the probability that j_1 has *x* free wavelengths is $q_{j_1}(x)$ and the probability that j_2 has *y* free wavelengths is $q_{j_2}(y)$. Given $q_{j_1}(x)$ and $q_{j_2}(y)$, the probability that there exist i common free wavelengths is denoted by given by $p(i \mid x, y)$ and it can be calculated as :

$$p(i \mid x, y) = \begin{cases} \beta(x, y, i), & i \le \min(x, y); x + y - i \le W; 1 \le x, y \le W \\ 0, & otherwise \end{cases} \qquad (10)$$

where

$$\beta(x, y, i) = \frac{\binom{y}{i}\binom{W-y}{x-i}}{\binom{W}{x}}. \qquad (11)$$

As such, $u_{S_k}(i)$ for a two-hop segment can be derived as:

$$u_{S_k}(i) = \sum_{x=0}^{W} \sum_{y=0}^{W} \left\{ p(i \mid x, y) \times q_{j_1}(x) \times q_{j_2}(y) \right\}. \qquad (12)$$

The above analysis can be extended to determine the $u_{S_k}(i)$ where the hop length of segment S_k, *h*, is more than 2. Suppose the link set of segment S_k is $\{j_1, j_2, \cdots, j_h\}$. We use S'_k to represent its sub-segment composed by links $\{j_1, j_2, \cdots, j_{h-1}\}$. By regarding segment S_k as the composition of sub-segment S'_k and link j_h, we can obtain $u_{S_k}(i)$ using the following recursive formula:

$$u_{S_k}(i) = \sum_{x=0}^{W} \sum_{y=0}^{W} \left\{ p(i \mid x, y) \times u_{S'_k}(x) \times q_{j_h}(y) \right\}. \qquad (13)$$

Now let us see how to calculate *P(X)*, the probability of conversion state *X* in route R_a. Suppose the probability that the *n*th WCR in route R_a has no free wavelength converter is p_n, we define

$$Y(n) = \begin{cases} 1 - p_n & \text{; the } n\text{th WCR has free converters} \\ p_n & \text{; the } n\text{th WCR has no free converter} \end{cases}. \qquad (14)$$

So we can have:

$$P(X) = \prod_{n=1}^{D} Y(n). \qquad (15)$$

Now the only unknown variable is p_n. For any accepted lightpath request bypassing the *n*th WCR, there are two different situations: (1) a common free wavelength exists on all links along that route; (2) there is no common free wavelength, but each segment has its own common free wavelength. Only in situation (2), wavelength converters are allocated in the WCRs, and we call the lightpaths which cause wavelength conversion as "conversion traffic". Let T_n denote the total "conversion traffic" bypassing the *n*th WCR. According to our wavelength assignment algorithm, an accepted lightpath uses wavelength conversion if and only if there is no common free wavelength on all links; only the lightpaths which use wavelength conversion are considered as conversion traffic. In addition, the probability that there is no common free wavelength on all links of R_a is $u_{R_a}(0)$, if we consider route R_a as a single segment. So T_n can be calculated as:

$$T_n = \sum_{\substack{\text{All the routes } \{R_a\} \text{ that} \\ \text{bypassing the } n\text{th WCR}}} \left\{ A_a (1 - B_{R_a}) u_{R_a}(0) \right\}. \qquad (16)$$

Assume the number of converters in the *n*th WCR is Z_n. We approximately consider that the conversion traffic arrives to the *n*th WCR following a Poisson process with rate T_n. Hence, it forms an $M/M/m/m$ system with Z_n servers. We can derive the probability p_n that the *n*th WCR has no free wavelength converter by:

$$p_n = \left[1 + \sum_{j=1}^{Z_n} \frac{\prod_{i=1}^{j} (Z_n - i + 1)}{T_n^j} \right]^{-1}. \qquad (17)$$

The numerical algorithm used to solve the above fixed-point non-linear equations is as follows:

1. Initialize B_{R_a} to 0 for all routes, and $q_j(0)$ to 0 for all links.
2. Determine α_j using Eq. (4) for all links.
3. Determine $q_j(m_j)$ using Eqs. (2) and (3) for all links.
4. Determine B_{R_a} for all routes using Eqs. (8)–(17). If the new values of B_{R_a} are converged to old ones, the iteration is terminated and we can go to Step (5). Otherwise go to Step (2) for next iteration.
5. Finally, determine the overall blocking probability B using Eq. (1).

C. Wavelength Converter Placement Problem

A very important problem in the SPWC network architecture is the placement of wavelength converters. Traditionally, this problem is defined in the context of Sparse Wavelength Conversion, that is, to determine a set of routing nodes with Complete Wavelength Conversion capability such that the overall network blocking probability can be minimized. In our SPWC network architecture, we should re-define the wavelength converter placement problem as two sub-problems: (1) how to find a set of nodes which will be placed with a WCR? (2) Given the total number of M converters, how to place them in the selected WCRs? We now propose a simulation-based scheme to solve these two sub-problems. Its performance is evaluated in Section IV.

The basic idea of our scheme is to conduct simulations assuming Full-Complete Wavelength Conversion; from the simulations, we can observe how many wavelength conversions are conducted in each node. Thus we obtain statistics on the following two parameters for each node n:

1) $A(n)$: the average number of busy converters

2) $P(n)$: the maximum number of busy converters

It's then straightforward to place more wavelength converters on the nodes with large values of $A(n)$ and $P(n)$. In the following, we use the 14-node NSFNET topology as an example to show how to choose the WCRs and assign wavelength converters to each WCR.

In this example, we assume each fiber link can support 40 wavelengths. In the simulations, 1,000,000 consecutive lightpath requests are generated. The total network traffic is 200 Erlangs and they are uniformly distributed to all the node pairs. The following routing and wavelength assignment algorithm is used to setup a lightpath:

Upon the arrival of a lightpath request:

1) Find the shortest path between the two end nodes of the lightpath request.

2) In the shortest path, if there exists any link that has no free wavelength, the lightpath request will be blocked.

3) If there exist common free wavelengths among all the links in the route, set up the lightpath by choosing the common free wavelength with the smallest label for each link.

4) Otherwise, for each link, use the first-fit wavelength assignment scheme. If two consecutive links use different wavelengths, a wavelength converter is allocated in the intermediate WCR.

We call the above wavelength assignment scheme in Steps 3 and 4 as *Modified First-Fit* (MFF) wavelength assignment. Step 3 is particularly important as we have shown in Section II-D that 90% of the bypassing lightpaths can be setup without wavelength conversion under low traffic. The simulation results are shown in Table 4.

Table 4 Conversion Statistics of NSFNET

Node n	*1*	*2*	*3*	*4*	*5*	*6*	*7*
A(n)	0.4	0.7	0.3	2.3	0.4	1.8	1.6
P(n)	9	12	9	22	11	19	16
Node n	*8*	*9*	*10*	*11*	*12*	*13*	*14*
A(n)	0	0.7	1.4	0.7	0.6	0	0.1
P(n)	0	13	16	12	11	0	6

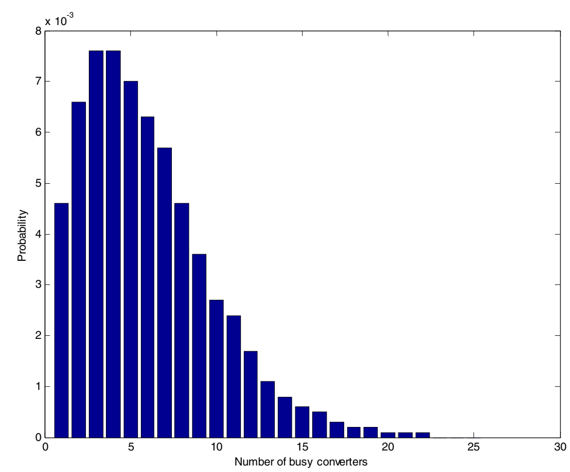


Fig. 7. Distribution of the number of busy wavelength converters at Node 4

From the values of $A(n)$ and $P(n)$ in Table 4, we can observe that the utilization of wavelength converters is indeed very low. Although the peak value of concurrently busy wavelength converters can be large, most of the time only a small fraction of converters are busy. To illustrate this, we show the probability distribution of the number of busy converters for node 4 in Fig. 7. Similar behavior can be observed in all the other nodes. It is worth noting that we do not plot the probability of no busy converter, which is about 93.5%.

The first observation is that most of the nodes have very little wavelength conversion activities. From Table 4, we can see that nodes 4, 6, 7, 10 have much more wavelength conversion activities than other nodes. The nodes 2, 9, 11, 12, 1, 5 and3 have less conversion activities. Nodes 8, 13 and 14 almost do not need wavelength conversion at all. These observations can provide some good reference for the network carriers to choose where to place the WCRs. As an example, we choose the set $K = \{4,6,7,10\}$ to place the WCRs. Then we can simply assign converters to the nodes in set K proportionally to the value of $A(n)$. For example, if there are 50 converters, the converter placement scheme is shown in Table 5.

Table 5 Proportional Placement Scheme for NSFNET, *M*=50

Node *n*	*4*	*6*	*7*	*10*
Number of converters	16	13	11	10

IV. NUMERICAL RESULTS AND ANALYSIS

In this section, we compare the performance of different conversion schemes for NSFNET topology (Fig. 4) and 25-node mesh-torus network topology (Fig. 9). As in many previous studies [5, 7, 10-13], we assume that the traffic is uniformly distributed to all node pairs. The lightpath requests arrive according to a Poisson process and the holding time is exponentially distributed with a unit time. We assume 40 wavelength channels are available for each fiber link.

A. Blocking Performance Analysis of NSFNET

Fig. 8 shows the blocking probabilities total network versus traffic load for different wavelength conversion schemes in NSFNET topology. In the simulations, we use the Shortest Path Routing and First-Fit wavelength assignment algorithm for the case with no wavelength conversion, and the RWA algorithm for sparse-partial wavelength conversion, as described in Section III-C.

The first observation from the figure is that, full-complete wavelength conversion can decrease the blocking probability by a large margin. The second significant result is that, compared to the 1,600 converters used in the full-complete wavelength conversion, only 50 converters are needed to achieve satisfactory performance if sparse-partial wavelength conversion schemes are used. The analytical results of the sparse-partial wavelength conversion are also presented. We notice that the analytical results of the blocking probability are larger than the simulation results. The first reason is that in the analytical model, the link traffic is modeled by Poisson distribution, which is conservative [7]. The second reason is that, the analytical model assumes random wavelength assignment is used. However, the simulations use the modified first-fit wavelength assignment algorithm, which can further decrease the blocking probability.

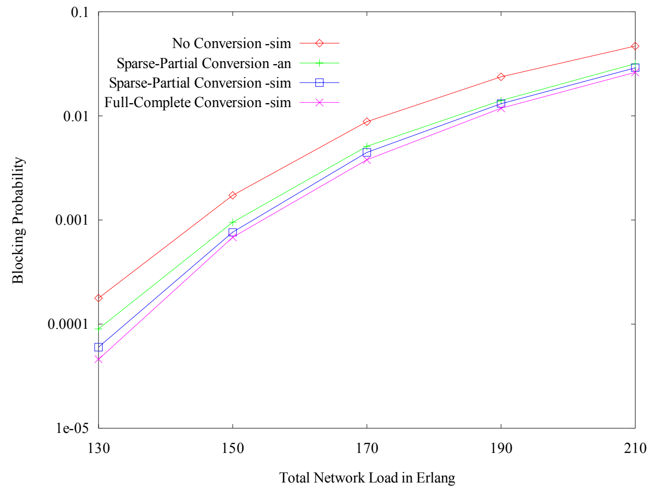


Fig. 8. Blocking Performance in NSFNET, *M* = 50

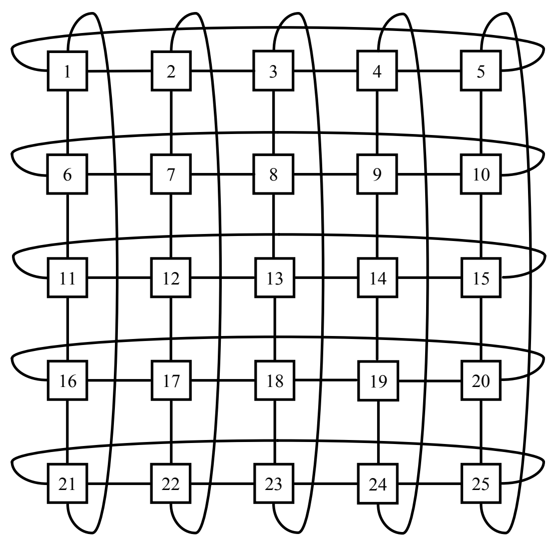


Fig. 9. 25-node Mesh-torus network

B. Blocking Performance Analysis of the Mesh-Torus Network Topology

In the mesh-torus network topology, full-complete wavelength conversion requires 25 WCRs and 4,000 wavelength converters. We follow the approach in Section III-B to place wavelength converters in the 25-node mesh-torus network. We first conduct simulations assuming full-complete wavelength conversion and get the conversion statistics. It turns out that each of the nodes 1-5 has much more wavelength conversion activities than the other 20 nodes. So we decide to use 5 WCRs to replace nodes 1-5. We also find that these 5 nodes have the same wavelength conversion activities. This is because of the because of the symmetry of the mesh-torus topology. Assume there are 75 wavelength converters; a straightforward placement scheme is to equip 15 converters for each WCR. We then conduct simulations for different conversion cases: no conversion, sparse-partial wavelength conversion with 5 WCRs where each WCR has 15 wavelength converters, and the full-complete wavelength conversion. The analytical results for sparse-partial wavelength conversion are also presented.

The results are shown in Fig. 10. We can observe that sparse-partial wavelength conversion works very well in mesh-torus topology. First, because of the effect of sparse conversion, 5 WCRs can achieve almost the same performance as 25 WCRs; second, because of the effect of partial conversion, only 15 wavelength converters for each WCR are needed to achieve almost the same performance of 160 wavelength converters. To conclude, only 75 wavelength converters are required for the 25-node mesh-torus network to achieve very close performance to that of 4,000 wavelength converters.

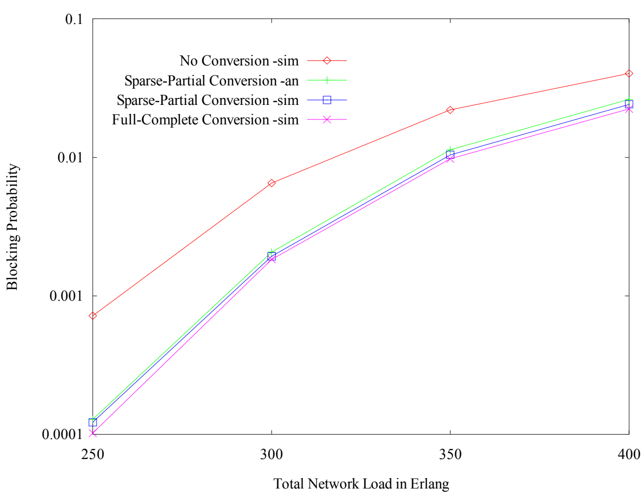


Fig. 10. Blocking Performance in 25-node mesh-torus network, $M = 75$

V. CONCLUSIONS

This paper addresses an important problem in wavelength-routed all-optical WDM networks: how to efficiently utilize a limited number of wavelength converters. We first explain why Partial Wavelength Conversion can achieve very close performance to Complete Wavelength Conversion. We then propose the Sparse-Partial Wavelength Conversion network architecture, which has the flexibility to install the partial WCRs gradually into the network. Both analytical and simulation results are presented. By using the proposed wavelength converter placement scheme and wavelength assignment algorithm, only a very small number of wavelength converters are needed to achieve very close performance to that of the Full-Complete Wavelength Conversion.

There are many possible future research directions within this framework. For example, in this paper, we assume that static shortest path routing is used. Given that many adaptive routing algorithms are effective in reducing blocking probability, it is possible to use them in our SPWC architecture. Wavelength converter placement and wavelength assignment under such advanced routing algorithms are also interesting issues worthy of further investigation.

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