Anonymity for Continuous Data Publishing[*]

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ABSTRACT

k-anonymization is an important privacy protection mechanism in data publishing. While there has been a great deal of work in recent years, almost all considered a single static release. Such mechanisms only protect the data up to the first release or first recipient. In practical applications, data is published continuously as new data arrive; the same data may be anonymized differently for a different purpose or a different recipient. In such scenarios, even when all releases are properly k-anonymized, the anonymity of an individual may be unintentionally compromised if recipient cross-examines all the releases received or colludes with other recipients. Preventing such attacks, called *correspondence attacks*, faces major challenges. In this paper, we systematically characterize the correspondence attacks and propose an efficient anonymization algorithm to thwart the attacks in the model of continuous data publishing.

1. INTRODUCTION

k-anonymization [11] is a promising approach to data publishing while protecting the identity of individuals. The data holder has a table of the form [1]

D(Explicit_identifier, Quasi_identifier, Sensitive_attribute).

Explicit_identifier consists of identifying information (such as SSN and names). Quasi_identifier (such as date of birth, gender, and zip code) does not reveal identity, but can be used to link to a person or an explicit identity in some external sources [11]. Sensitive_attribute consists of other person-specific information (such as medical and DNA entries). Instead of publishing the original table D, the data holder publishes an anonymized release

$$R(QID, \text{Sensitive\_attribute}),$$

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where QID is a *k-anonymized* version of Quasi_identifier [11]: each record belongs to an equivalence class of size at least k and all records in an equivalence class are made indistinguishable with respect to QID by hiding some details. In a k-anonymized release, if an individual is linked to a record through QID, it is also linked to at least $k-1$ other records. Sensitive_attribute is not modified because the data usefulness depends on the exact information of this attribute.

k-anonymization has been primarily studied for a single static release of data [3, 4, 6, 9, 11, 13]. In practice, however, new data arrives continuously and up-to-date data has to be published to researchers in a timely manner. One approach is to anonymize and publish new records *separately* each time they arrive. This naive approach suffers from severe data distortion because small increments are anonymized independently. Moreover, it is difficult to analyze a collection of independently anonymized data sets. For example, if the country name (e.g., Canada) is used in the release for first month records and if the city name (e.g., Toronto) is used in the release for second month records, counting the total number of persons born in Toronto is not possible. Another approach is to enforce the later release to be no more specialized than the previous releases [13]. The major drawback is that each subsequent release gets increasingly distorted, even if more new data are available.

1.1 A Practical Publishing Model

Suppose that the data holder previously collected a set of records D_1 timestamped T_1, and published a k-anonymized version of D_1, denoted release R_1. Then the data holder collects a new set of records D_2 timestamped T_2 and wants to publish a k-anonymized version of all records collected so far, $D_1 \cup D_2$, denoted release R_2. Note, D_i contains the "events" that happened at time T_i. An event, once occurred, becomes part of the history, therefore, cannot be deleted. This *publishing scenario* is different from *update scenario* in standard data management where deletion of records can occur. R_i simply publishes the "history", i.e., the events that happened up to time T_i. A real-life example can be found in California where the hospitals are required to submit specific demographic data of all discharged patients every six months.[1] The above publishing model directly serves the following scenarios.

Continuous data publishing. Publishing the release R_2 for $D_1 \cup D_2$ would permit an analysis on the data over the combined time period of T_1 and T_2. It also takes the advantage of data abundance over a longer period of time

[1]http://www.oshpd.ca.gov/HQAD/PatientLevel/

to reduce data distortion required by anonymization.

Multi-purpose publishing. With D_2 being empty, R_1 and R_2 can be two releases of D_1 anonymized differently to serve different information needs, such as correlation analysis vs clustering analysis, or different recipients, such as a medical research team vs a health insurance company, who may collude together by sharing their received data.

We first describe the publishing model with two releases and then show the extension beyond two releases in Section 7. Following the convention of k-anonymity [13], we assume that each individual has at most one record in $D_1 \cup D_2$. This assumption holds in many real-life databases. For example, in a normalized customer data table, each customer has only one profile. In the case that an individual has a record in both D_1 and D_2, there will be two duplicates in $D_1 \cup D_2$ and one of them can be removed in a preprocessing. As in [17], we assume that the records of an individual remain the same in D_1 and D_2.

1.2 Correspondence Attacks

We show, by an example, that the traditional k-anonymization is insufficient for preventing correspondence attacks.

EXAMPLE 1.1. Consider Tables 1-2, where QID is [Birthplace,Job] and the sensitive attribute is Disease. The data holder (e.g., a hospital) published the 5-anonymized R_1 for 5 records a_1-a_5 collected in the previous month (i.e., timestamp T_1). The anonymization was done by generalizing UK and France into Europe; the original values in the brackets are not released. In the current month (i.e., timestamp T_2), the data holder collects 5 new records (i.e., b_6-b_{10}) and publishes the 5-anonymized R_2 for all 10 records collected so far. Records are shuffled to prevent mapping between R_1 and R_2 by their order. The recipients know that every record in R_1 has a "corresponding record" in R_2 (that originates from the same record in D_1). One recipient, the *attacker*, tries to identify his neighbor Alice's record from R_1 or R_2, knowing that Alice was admitted to the hospital, as well as, Alice's QID and timestamp. Consider the following scenarios.

Scenario I. Alice has QID=[France, Lawyer] and timestamp T_1. The attacker seeks to identify Alice's record in R_1. Examining R_1 alone, Alice's QID matches all 5 records in R_1. However, examining R_1 and R_2 *together*, the attacker learns that the records a_1,a_2,a_3 cannot all originate from Alice's QID; otherwise R_2 would have contained at least three records of [France, Professional, Flu] since every record in R_1 has a corresponding record in R_2. Consequently, the attacker excludes one of a_1,a_2,a_3 as possibility; the choice among a_1,a_2,a_3 does not matter as they are identical.

Scenario II. Alice has QID=[France, Lawyer] and timestamp T_1. Knowing that R_2 contains all records at T_1 and T_2, the attacker seeks to identify Alice's record in R_2. The attacker infers that, among the matching records b_4-b_8 in R_2, at least one of b_4,b_5,b_6 must have timestamp T_2; otherwise b_4,b_5,b_6 would have timestamp T_1, in which case there would have been at least three (corresponding) records of the form [Europe, Lawyer, HIV] in R_1. In this case, the attacker excludes at least one of b_4,b_5,b_6 since Alice has timestamp T_1.

Scenario III. Alice has QID=[UK, Lawyer] and timestamp T_2 and the attacker seeks to identify Alice's record in R_2. The attacker infers that, among the matching records b_1,b_2,b_3,b_9,b_{10} in R_2, at least one of b_1,b_2,b_3 must have timestamp T_1; otherwise one of a_1,a_2,a_3 would have no corresponding record in R_2. In this case, at least one of b_1,b_2,b_3 is excluded since Alice has timestamp T_2.

Table 1: 5-anonymized R_1

RID	Birthplace	Job	Disease
(a_1)	Europe (UK)	Lawyer	Flu
(a_2)	Europe (UK)	Lawyer	Flu
(a_3)	Europe (UK)	Lawyer	Flu
(a_4)	Europe (France)	Lawyer	HIV
(a_5)	Europe (France)	Lawyer	HIV

Table 2: 5-anonymized R_2

RID	Birthplace	Job	Disease
(b_1)	UK	Professional (Lawyer)	Flu
(b_2)	UK	Professional (Lawyer)	Flu
(b_3)	UK	Professional (Lawyer)	Flu
(b_4)	France	Professional (Lawyer)	HIV
(b_5)	France	Professional (Lawyer)	HIV
(b_6)	France	Professional (Lawyer)	HIV
(b_7)	France	Professional (Doctor)	Flu
(b_8)	France	Professional (Doctor)	Flu
(b_9)	UK	Professional (Doctor)	HIV
(b_{10})	UK	Professional (Lawyer)	HIV

In each scenario, at least one matching record is excluded, so the 5-anonymity of Alice is compromised. ∎

All these attacks "crack" some matching records in R_1 or R_2 by inferring that they either do not originate from Alice's QID or do not have Alice's timestamp. Such cracked records are not related to Alice, thus, excluding them allows the attacker to focus on a smaller set of candidates. Since cracked records are identified by cross-examining R_1 and R_2 and by exploiting the knowledge that every record in R_1 has a "corresponding record" in R_2, such attacks are called *correspondence attacks*.

Having access to only R_1 and R_2, not D_1 and D_2, cracking a record is not straightforward. For example, to crack a record in R_1 for Alice having QID=[France, Lawyer], the attacker must show that the *original* birthplace in the record is not France, whereas the published Europe may or may not originate from France. Similarly, it is not straightforward to infer the timestamp of a record in R_2. For example, any three of b_1,b_2,b_3,b_7,b_8 can be the corresponding records of a_1,a_2,a_3, so none of them must have timestamp T_1. In fact, observing only the published records, there are many possible assignments of corresponding records between R_1 and R_2. For example, one assignment is

(a_1, b_1), (a_2, b_2), (a_3, b_3), (a_4, b_4), (a_5, b_5),

where the original record represented by (a_1, b_1) is [UK, Lawyer, Flu]. Another assignment is

(a_1, b_7), (a_2, b_2), (a_3, b_8), (a_4, b_6), (a_5, b_9).

In this assignment, the original record represented by (a_1, b_7) is [France, Lawyer, Flu]. All such assignments are possible to the attacker because they all produce the same "view", i.e., R_1 and R_2. Detecting correspondence attacks assuming this view of the attacker is non-trivial.

1.3 Contributions

In this paper, we formalize the notion of correspondence attacks and present an approach to prevent such attacks. We focus on answering several key questions:

- *Given that there are many possible ways of assigning corresponding pairs, and each may lead to a different inference, what should the attacker assume while cracking a record?* We present a model of correspondence attacks to address this issue and show that the continuous data publishing problem subsumes the case with multiple colluding recipients (Section 3).

- *What are exactly the records that can be cracked based on R_1 and R_2?* We systematically characterize the set of cracked records by correspondence attacks and propose the notion of *BCF-anonymity* to measure anonymity assuming this power of the attacker (Section 4).
- *Can R_2 be anonymized such that R_2 satisfies BCF-anonymity yet remains useful?* We show that the optimal BCF-anonymization is NP-hard. Then, we develop a practically efficient algorithm to determine a BCF-anonymized R_2 (Section 5), study its data quality (Section 6), and extend the proposed approach to deal with more than two releases and other privacy notions (Section 7).

2. RELATED WORK

Anonymizing continuous data is a challenging problem because the attacker has increasingly more knowledge about the previously published data. Some anonymization methods [2, 10, 14, 17] have been proposed recently.

[2] is an early study which investigates the continuous data publishing scenario, but the anonymization method relies on delaying records release and the delay can be unbounded. Consequently, records could not be released in a timely fashion. In our method, records collected at timestamp T_i are *always* published in the corresponding release R_i without delay. Moreover, [2] guarantees a relatively weak privacy notion, which requires each equivalence class to contain at least ℓ distinct sensitive values. This privacy notion is not safe if some sensitive values dominate an equivalence class. In contrast, our proposed approach can achieve anonymity, *confidence bounding* [15], *(α,k)-anonymity* [16], and other safer notions of *ℓ-diversity* [8].

[17] presents the first study to address both record insertions and deletions in data re-publication. It proposes a new privacy notion called *m-invariance*: if a record r has been published in releases $R_i, \cdots, R_j$ where $i < j$, then all QID groups containing r must have the same set of sensitive values, called the *signature* [17]. This will ensure the intersection of sensitive values over all such groups does not reduce the set of sensitive values. To maintain m-invariance, their method adds "counterfeit records" corresponding to infrequent sensitive values to make those equivalence classes have the same signature. Counterfeits, however, may not be acceptable in some cases. Suppose a pharmaceutical company wants to analyze patient reaction to certain drugs. Infrequent sensitive values such as the negative reactions are the most interesting ones and the target for research. However, with many counterfeit negative reactions which correspond to no real-life entities, it is difficult to deploy the results obtained from such data. Note that, even in the "insertion only" case, adding counterfeits is still necessary, for example, when a record with a new sensitive value is added. In contrast, our method guarantees data truthfulness at record level: each published record corresponds to a real-life entity.

Our previous work [14] studies the problem of anonymizing sequential releases where each subsequent release publishes a different subset of attributes for the same set of records. In contrast, this paper considers each release that combines new records with previously collected records over the same set of attributes. The attack and prevention mechanisms are very different in these two publishing models.

[10] considers the scenario that the Case-ID of records must be published. In our work, we consider the scenario that the data holder has removed the Case-ID of records, so the attack based on Case-ID [10] does not occur. Instead, we deal with a new type of attacks even if no Case-ID is published. [5] proposes an efficient index structure to incrementally k-anonymize each individual release, but it does not address the correspondence attacks studied in this paper.

3. PROBLEM STATEMENTS

Every attribute has a finite set of *domain values*. Numeric values are pre-discretized into intervals. A *raw record* contains only domain values. Every attribute in QID has a taxonomy (tree) in which leaf nodes represent domain values and high level values generalize low level values. To generalize a table, we employ the global recoding scheme, i.e., globally replace all instances of some domain values with their ancestor value in the taxonomy of some attributes A_j in QID. A generalized attribute A_j can be represented by a "cut" through its taxonomy tree. A cut of a tree is a subset of values in the tree which contains exactly one value on each root-to-leaf path [4]. R_1 and R_2 in Tables 1-2 are examples.

There are some other generalization schemes, such as multidimensional [7] and local recoding [10, 16, 17], that cause less data distortion, but these schemes make data analysis difficult. In local recoding, for example, some instances of Lawyer can be generalized to Professional while some instances of Lawyer remain ungeneralized. As a result, counting the number of lawyers becomes impossible because some lawyers are represented by professionals. Furthermore, many standard data mining methods, e.g., decision tree analysis, treat Lawyer and Professional as two independent values. Consequently, the decision tree may contain two branches, (Lawyer → class1) and (Professional → class2). It is unclear which branch should be used to classify a new lawyer.

In a *k-anonymized* table, records are partitioned into equivalence classes of size (i.e., the number of records) at least k. Each equivalence class contains all records having the same value on QID. We use qid to denote both a value on QID and the corresponding equivalence class. $|qid|$ denotes the size of the equivalence class. A *group g* in an equivalence class qid consists of the records in qid that have the same value on the sensitive attribute. In other words, a group contains all records in the table that are indistinguishable wrt QID and the sensitive attribute. A person *matches* a (generalized) record in a table if her QID is either equal to or more specific than the record on every attribute in QID.

3.1 Correspondence Attacks

The data holder previously collected some data D_1 timestamped T_1 and published a k-anonymized version of D_1, called release R_1. Then the data holder collects new data D_2 timestamped T_2 and publishes a k-anonymized version of $D_1 \cup D_2$, called release R_2. An *attacker*, one of the recipients of R_1 and R_2, attempts to identify the record of some *target* person, denoted by P, from R_1 or R_2. We assume that the attacker is aware of P's QID and timestamp. In addition, the attacker has the following *correspondence knowledge*: (1) Every record timestamped T_1 (i.e., from D_1) has a record in R_1 and a record in R_2, called *corresponding records.* (2) Every record timestamped T_2 (i.e., from D_2) has a record in R_2, but not in R_1. Below is an intuition of the three possible attacks based on such knowledge.

Forward-attack, denoted by F-attack(R_1, R_2). P has timestamp T_1 and the attacker tries to identify P's record in the *cracking release* R_1 using the *background release* R_2. Since P has a record in R_1 and a record in R_2, if a matching record r_1 in R_1 represents P, there must be a corresponding record in R_2 that matches P's QID and agrees with r_1 on the sensitive attribute. If r_1 fails to have such a corresponding record in R_2, then r_1 does not originate from P's QID, and therefore, r_1 can be excluded from the possibility of P's record. Scenario I is an example.

Cross-attack, denoted by C-attack(R_1, R_2). P has timestamp T_1 and the attacker tries to identify P's record in the *cracking release* R_2 using the *background release* R_1. Similar to F-attack, if a matching record r_2 in R_2 represents P, there must be a corresponding record in R_1 that matches P's QID and agrees with r_2 on the sensitive attribute. If r_2 fails to have such a corresponding record in R_1, then r_2 either has timestamp T_2 or does not originate from P's QID, and therefore, r_2 can be excluded from the possibility of P's record. Scenario II is an example.

Backward-attack, denoted by B-attack(R_1, R_2). P has timestamp T_2 and the attacker tries to identify P's record in the *cracking release* R_2 using the *background release* R_1. In this case, P has a record in R_2, but not in R_1. Therefore, if a matching record r_2 in R_2 has to be the corresponding record of some record in R_1, then r_2 has timestamp T_1, and therefore, r_2 can be excluded from the possibility of P's record. Scenario III is an example. Note that it is impossible to single out the matching records in R_2 that have timestamp T_2 but do not originate from P's QID since all records at T_2 have no corresponding record in R_1.

Table 3 summarizes all four possible combinations of cracking release (R_1 or R_2) and target P's timestamp (T_1 or T_2). Note that if a target P has timestamp T_2, P does not have a record in R_1, so it is impossible to crack P's record in R_1 in such a case and there are only three types of attacks.

All these attacks are based on making some inferences about corresponding records. There are many possible assignments of corresponding records and each assignment implies a possibly different underlying data (D'_1, D'_2), not necessarily *the* underlying data (D_1, D_2) collected by the data holder. Since all such underlying data (D'_1, D'_2) generate the same published R_1 and R_2, they are all possible to the attacker who knows about the data only through the published R_1 and R_2. This observation motivates us to consider only the inferences that do not depend on a particular choice of a candidate (D'_1, D'_2). First, let us define the space of such candidates underlying data for the published R_1 and R_2.

Consider a record r in R_1 or R_2. An *instantiation* of r is a raw record that agrees with r on the sensitive attribute and specializes r or agrees with r on QID. A *generator* of (R_1, R_2) is an assignment, denoted by I, from the records in $R_1 \cup R_2$ to their instantiations such that: for each record r_1 in R_1, there is a distinct record r_2 in R_2 such that $I(r_1) = I(r_2)$; (r_1, r_2) is called *buddies* under I. Duplicate records are treated as distinct records. The buddy relationship is *injective*: no two records have the same buddy. Every record in R_1 has a buddy in R_2 and exactly $|R_1|$ records in R_2 have a buddy in R_1. If r_2 in R_2 has a buddy in R_1, $I(r_2)$ has timestamp T_1; otherwise $I(r_2)$ has timestamp T_2. Intuitively, a generator represents an underlying data for (R_1, R_2) and each pair of buddies represent corresponding records in the generator.

EXAMPLE 3.1. Refer to R_1 and R_2 in Tables 1-2. One generator has the buddies:

Table 3: Types of correspondence attacks

	Target P's timestamp	Cracking release	Background release
F-attack	T_1	R_1	R_2
C-attack	T_1	R_2	R_1
B-attack	T_2	R_2	R_1
No attack	T_2	R_1	R_2

(a_1, b_1), assigned to [UK, Lawyer, Flu]
(a_2, b_2), assigned to [UK, Lawyer, Flu]
(a_3, b_3), assigned to [UK, Lawyer, Flu]
(a_4, b_4), assigned to [France, Lawyer, HIV]
(a_5, b_5), assigned to [France, Lawyer, HIV].

$I(b_1)$-$I(b_5)$ have timestamp T_1. b_6-b_{10} can be assigned to any instantiation. Another generator has the buddies:

(a_1, b_7), assigned to [France, Lawyer, Flu]
(a_2, b_2), assigned to [UK, Lawyer, Flu]
(a_3, b_8), assigned to [France, Lawyer, Flu]
(a_4, b_6), assigned to [France, Lawyer, HIV]
(a_5, b_9), assigned to [UK, Lawyer, HIV].

$I(b_2)$, $I(b_6)$-$I(b_9)$ have timestamp T_1. Note that these generators give different underlying data. ■

Consider a record r in R_1 or R_2. Suppose that for *some* generator I, the instantiation $I(r)$ matches P's QID and timestamp. In this case, excluding r means information loss for the purpose of attack because there is some underlying data for (R_1, R_2) (given by I) in which r is P's record. On the other hand, suppose that for *no* generator I the instantiation $I(r)$ can match P's QID and timestamp. Then r *definitely* cannot be P's record, so excluding r losses no information to the attacker. Our attack model is based on excluding such non-representing records.

For a target P with timestamp T_1, if P has a record in R_1, P must match some qid_1 in R_1 and some qid_2 in R_2. Therefore, we assume such a matching pair (qid_1, qid_2) for P. Recall that a group g_i in an equivalence class qid_i consists of the records in qid_i that agree on the sensitive attribute. For a generator I, $I(g_i)$ denotes the set of records $\{I(r_i) \mid r_i \in g_i\}$. Below, we present the formal definition for each type of attacks. r_i, g_i, qid_i refer to records, groups and equivalence classes from R_i, $i = 1, 2$.

3.1.1 F-attack

The F-attack seeks to identify as many as possible records in an equivalence class qid_1 that do not represent P in *any* choice of the generator. Such *cracked records* definitely cannot be P's record, and therefore, can be excluded. Since all records in a group are identical, the choice of cracked records in a group does not make a difference and determining the number of cracked records (i.e., the crack size) is sufficient to define the attack.

DEFINITION 3.1 (CRACK SIZE). Assume that a target P has timestamp T_1 and matches (qid_1, qid_2). A group g_1 in qid_1 has *crack size* c wrt P if c is maximal such that for every generator I, at least c records in $I(g_1)$ do not match P's QID. ■

If g_1 has crack size c, at least c records in g_1 can be excluded from the possibility of P's record. On the other hand, with c being maximal, excluding more than c records will result in excluding some record that can possibly be P's record. Therefore, the crack size is both the minimum and the maximum number of records that can be excluded from g_1 without any information loss for the purpose of attack.

EXAMPLE 3.2. Consider Scenario I in Example 1.1. Alice has QID=[France, Lawyer] and matches (qid_1, qid_2), where
qid_1 = [Europe, Lawyer] = $\{a_1, a_2, a_3, a_4, a_5\}$
qid_2 = [France, Professional] = $\{b_4, b_5, b_6, b_7, b_8\}$.
qid_1 has two groups: $g_1 = \{a_1, a_2, a_3\}$ for Flu and $g'_1 = \{a_4, a_5\}$ for HIV. g_1 has crack size 1 wrt Alice because, for any generator I, at least one of $I(a_1)$,$I(a_2)$,$I(a_3)$ does not match Alice's QID: if all of $I(a_1)$,$I(a_2)$,$I(a_3)$ match Alice's QID, R_2 would have contained three buddies of the form [France, Professional, Flu]. The crack size is maximal since the second generator I in Example 3.1 shows that only one of $I(a_1)$,$I(a_2)$,$I(a_3)$ (i.e., $I(a_2)$) does not match Alice's QID. ■

Definition 3.1 does not explain *how* to effectively determine the crack size, which is the topic in Section 4. For now, assuming that the crack size is known, we want to measure the anonymity after excluding the cracked records from each equivalence class. The F-anonymity below measures the minimum size of an equivalence class in R_1 after excluding all records cracked by F-attack.

DEFINITION 3.2 (F-ANONYMITY). Let $F(P, qid_1, qid_2)$ be the sum of the crack sizes for all groups in qid_1 wrt P. $F(qid_1, qid_2)$ denotes the maximum $F(P, qid_1, qid_2)$ for any target P that matches (qid_1, qid_2). $F(qid_1)$ denotes the maximum $F(qid_1, qid_2)$ for all qid_2 in R_2. The *F-anonymity* of (R_1, R_2), denoted by $FA(R_1, R_2)$ or FA, is the minimum $(|qid_1| - F(qid_1))$ for all qid_1 in R_1. ■

3.1.2 C-attack

DEFINITION 3.3 (CRACK SIZE). Assume that a target P has timestamp T_1 and matches (qid_1, qid_2). A group g_2 in qid_2 has *crack size* c wrt P if c is maximal such that for every generator I, at least c records in $I(g_2)$ do not match either P's timestamp or P's QID. ■

EXAMPLE 3.3. Consider Scenario II in Example 1.1. Alice has QID=[France, Lawyer] and matches $(qid_1$,$qid_2)$ where
qid_1 = [Europe, Lawyer] = $\{a_1, a_2, a_3, a_4, a_5\}$
qid_2 = [France, Professional] = $\{b_4, b_5, b_6, b_7, b_8\}$.
qid_2 has two groups: $g_2 = \{b_7, b_8\}$ for Flu, and $g'_2 = \{b_4, b_5, b_6\}$ for HIV. g'_2 has crack size 1 wrt Alice since, for any generator I at least one of $I(b_4)$,$I(b_5)$,$I(b_6)$ does not match Alice's timestamp or QID; otherwise, R_1 would have contained three buddies of the form [Europe, Lawyer, HIV]. The crack size is maximal since the first generator I in Example 3.1 shows that $I(b_4)$ and $I(b_5)$ match Alice's timestamp and QID. ■

DEFINITION 3.4 (C-ANONYMITY). Let $C(P, qid_1, qid_2)$ be the sum of crack sizes of all groups in qid_2 wrt P. $C(qid_1, qid_2)$ denotes the maximum $C(P, qid_1, qid_2)$ for any target P that matches (qid_1, qid_2). $C(qid_2)$ denotes the maximum $C(qid_1, qid_2)$ for all qid_1 in R_1. The *C-anonymity of* (R_1, R_2), denoted by $CA(R_1, R_2)$ or CA, is the minimum $(|qid_2| - C(qid_2))$ for all qid_2 in R_2. ■

3.1.3 B-attack

A target P for B-attack has timestamp T_2, thus, does not have to match any qid_1 in R_1.

DEFINITION 3.5 (CRACK SIZE). Assume that a target P has timestamp T_2 and matches qid_2 in R_2. A group g_2 in qid_2 has *crack size* c wrt P if c is maximal such that for every generator I, at least c records in $I(g_2)$ have timestamp T_1. ■

EXAMPLE 3.4. Consider Scenario III in Example 1.1. Alice has timestamp T_2 and QID=[UK, Lawyer]. qid_2=[UK, Professional] consists of $g_2 = \{b_1, b_2, b_3\}$ for Flu and $g'_2 = \{b_9, b_{10}\}$ for HIV. g_2 has crack size 1 wrt Alice. For every generator I, at least one of $I(b_1)$,$I(b_2)$,$I(b_3)$ has timestamp T_1; otherwise, one of a_1,a_2,a_3 would have no buddy in R_2. The crack size is maximal since the second generator I in Example 3.1 shows that only $I(b_2)$, among $I(b_1)$,$I(b_2)$,$I(b_3)$, has timestamp T_1. ■

DEFINITION 3.6 (B-ANONYMITY). Let $B(P, qid_2)$ be the sum of the crack sizes of all groups in qid_2 wrt P. $B(qid_2)$ denotes the maximum $B(P, qid_2)$ for any target P that matches qid_2. The *B-anonymity* of (R_1, R_2), denoted by $BA(R_1, R_2)$ or BA, is the minimum $(|qid_2| - B(qid_2))$ for all qid_2 in R_2. ■

3.2 Two Problems

A *BCF-anonymity requirement* states that *all* of BA, CA and FA are equal to or larger than some data-holder-specified threshold. We study two problems. The first problem checks whether a BCF-anonymity requirement is satisfied, assuming the input (i.e., R_1 and R_2) as viewed by the attacker.

DEFINITION 3.7 (DETECTION). Given R_1 and R_2, as described above, the *BCF-detection problem* is to determine whether a BCF-anonymity requirement is satisfied. ■

The second problem is to produce a generalized R_2 that satisfies a given BCF-anonymity requirement and remains useful. This problem assumes the input as viewed by the data holder, that is, R_1, D_1 and D_2, and uses an information metric to measure the usefulness of the generalized R_2. Examples are *discernibility cost* [12] and *data distortion* [11].

DEFINITION 3.8 (ANONYMIZATION). Given R_1, D_1 and D_2, as described above, the *BCF-anonymization problem* is to generalize $R_2 = D_1 \cup D_2$ so that R_2 satisfies a given BCF-anonymity requirement and remains as useful as possible wrt a specified information metric. ■

In the special case of empty D_1, F-attack and C-attack do not happen and B-anonymity coincides with k-anonymity of R_2 for D_2. Since the optimal k-anonymization is NP-hard [9], the optimal BCF-anonymization is NP-hard.

So far, we have assumed that both R_1 and R_2 are received by one recipient. In the special case of empty D_2, R_1 and R_2 are two different generalized versions of the same data D_1 to serve different information requirements or different recipients. In this case, there are potentially multiple attackers. What happens if the attackers collude together? The collusion problem may seem to be very different. Indeed, Definitions 3.7-3.8 subsume the collusion problem. Consider the worst-case collusion scenario in which *all* recipients collude together by sharing *all* of their received data. This scenario is equivalent to publishing all releases to one attacker.

4. DETECTION

The key to the BCF-detection problem is computing the crack size in Definitions 3.1, 3.3, 3.5. We present a method for computing the crack size of a group. Our insight is that if a record r represents the target P for some generator, its buddy in the other release (i.e., the corresponding record) must satisfy some conditions. If such conditions fail, r does not represent P for that generator. One of the conditions is the following "comparable" relationship.

Definition 4.1. For qid_1 in R_1 and qid_2 in R_2, (qid_1, qid_2) are *comparable* if for every attribute A in QID, $qid_1[A]$ and $qid_2[A]$ are on the same path in the taxonomy of A. For a record in r_1 (or a group) in qid_1 and a record r_2 (or a group) in qid_2, (r_1, r_2) are *comparable* if they agree on the sensitive attribute and (qid_1, qid_2) are comparable. For comparable (qid_1, qid_2), $CG(qid_1, qid_2)$ denotes the set of group pairs $\{(g_1, g_2)\}$, where g_1 and g_2 are groups in qid_1 and qid_2 for the same sensitive value and there is one pair (g_1, g_2) for each sensitive value (unless both g_1 and g_2 are empty). ■

Essentially, being comparable means sharing a common instantiation. For example, qid_1 = [Europe, Lawyer] and qid_2 = [UK, Professional] are comparable, but qid_1 = [Europe, Lawyer] and qid_2 = [Canada, Professional] are not. It is easy to see that if a target P matches (qid_1, qid_2), (qid_1, qid_2) are comparable; if (r_1, r_2) are buddies (for some generator), (r_1, r_2) are comparable; comparable (r_1, r_2) can be assigned to be buddies (because of sharing a common instantiation). The following fact can be verified:

Theorem 4.1. Suppose that P matches (qid_1, qid_2) and that (r_1, r_2) are buddies for a generator I. If $I(r_1)$ and $I(r_2)$ match P's QID, then r_1 is in g_1 if and only if r_2 is in g_2, where (g_1, g_2) is in $CG(qid_1, qid_2)$. ■

Theorem 4.1 follows becuase buddies agree on the sensitive attribute and $I(r_1)$ and $I(r_2)$ matching P's QID implies that r_1 is in qid_1 and r_2 is in qid_2. The next two lemmas are used to derive an upper bound on crack size. The first states some transitivity of the "comparable" relationship, which will be used to construct a required generator in the upper bound proof.

Lemma 4.1 (3-hop transitivity). Let r_1, r_1' be in R_1 and r_2, r_2' be in R_2. If each of (r_1', r_2), (r_2, r_1) and (r_1, r_2') is comparable, (r_1', r_2') is comparable.

Proof. The three comparable pairs imply that r_1' and r_2' agree on the sensitive attribute. We show that $r_1'[A]$ and $r_2'[A]$ are on the same path of the taxonomy for every attribute A in QID. Recall that the generalization in A forms a cut of the taxonomy of A. Therefore, with both r_1 and r_1' being comparable to r_2, either $r_1[A] = r_1'[A]$ or both are more specific than $r_2[A]$. Similarly, either $r_2[A] = r_2'[A]$ or both are more specific than $r_1[A]$. At least one of these equalities must hold; otherwise $r_1[A]$ is both more general than and more specific than $r_2[A]$, which is impossible. If $r_1[A] = r_1'[A]$ holds, the proof is done because $r_1[A]$ and $r_2'[A]$ are on the same path. If $r_2[A] = r_2'[A]$ holds, the proof is done because $r_1'[A]$ and $r_2[A]$ are on the same path. □

The following is the key lemma for proving the upper bound of crack size.

Lemma 4.2. Let (g_1, g_2) be in $CG(qid_1, qid_2)$. There exists a generator in which exactly $min(|g_1|, |g_2|)$ records in g_1 have a buddy in g_2.

Proof. Consider any generator I. Since the buddy relationship is injective, at most $min(|g_1|, |g_2|)$ records in g_1 have a buddy in g_2. Let X contain all such records and possibly other records in g_1 so that $|X| = min(|g_1|, |g_2|)$. For each r_1 in X that has a buddy r_2' not in g_2, we create a new buddy r_2 that is in g_2. Since less than $min(|g_1|, |g_2|)$ records in g_1 have a buddy in g_2, some record r_2 in g_2 does not have a buddy in g_1. If r_2 has no buddy, let r_2 be the new buddy of r_1. Assume that r_2 has a buddy r_1'. Note that r_1' is not in g_1. Each of (r_1', r_2), (r_2, r_1) and (r_1, r_2') is comparable (because buddies are comparable). From Lemma 4.1, (r_1', r_2') is comparable. Therefore, we can swap the buddies of r_1 and r_1', that is, make (r_1, r_2) and (r_1', r_2') the new buddies. Note that every record in X that previously has a buddy in g_2 is not affected by the swapping. □

Now we determine the crack size for each type of attack.

4.1 F-attack

Assume that P matches (qid_1, qid_2). Consider a group pair (g_1, g_2) in $CG(qid_1, qid_2)$. Since the buddy relationship is injective, if g_1 contains more records than g_2, i.e., $|g_1| > min(|g_1|, |g_2|)$, at least $|g_1| - min(|g_1|, |g_2|)$ records in g_1 do not have a buddy in g_2 for *any* generator. According to Theorem 4.1, these records do not originate from P's QID for any generator. Thus the crack size of g_1 is at least $|g_1| - min(|g_1|, |g_2|)$ (i.e., a lower bound). On the other hand, according to Lemma 4.2, there exists *some* generator in which *exactly* $|g_1| - min(|g_1|, |g_2|)$ records in g_1 do not have a buddy in g_2; according to Theorem 4.1, these records do not originate from P's QID for any generator. By Definition 3.1, $|g_1| - min(|g_1|, |g_2|)$ is the crack size of g_1.

Theorem 4.2. Suppose that a target P matches (qid_1, qid_2). Let (g_1, g_2) be in $CG(qid_1, qid_2)$. (1) g_1 has crack size c wrt P, where $c = |g_1| - min(|g_1|, |g_2|)$. (2) $F(qid_1, qid_2) = \sum c$, where $\sum$ is over (g_1, g_2) in $CG(qid_1, qid_2)$ and g_1 has the crack size c determined in (1). ■

Remarks. $F(P, qid_1, qid_2)$ is the same for all targets P that match (qid_1, qid_2), i.e., $F(qid_1, qid_2)$ computed by Theorem 4.2. To compute FA, we compute $F(qid_1, qid_2)$ for all comparable (qid_1, qid_2). This requires partitioning the records into equivalence classes and groups, which can be done by sorting the records on all attributes. The F-attack happens when $|g_1| > min(|g_1|, |g_2|)$, that is, g_1 contains too many records for their buddies to be contained in g_2. This could be the case if R_2 has less generalization due to additional records at timestamp T_2.

Example 4.1. Continue with Example 3.2. Alice matches qid_1=[Europe, Lawyer] and qid_2=[France, Professional]. qid_1 consists of $g_1 = \{a_1, a_2, a_3\}$ for Flu and $g_1' = \{a_4, a_5\}$ for HIV. qid_2 consists of $g_2 = \{b_7, b_8\}$ for Flu and $g_2' = \{b_4, b_5, b_6\}$ for HIV. $CG(qid_1, qid_2) = \{(g_1, g_2), (g_1', g_2')\}$. $|g_1| = 3$, $|g_2| = 2$, $|g_1'| = 2$ and $|g_2'| = 3$. So g_1 has crack size 1 and g_1' has crack size 0. ■

4.2 C-attack

By a similar argument, at least $|g_2| - min(|g_1|, |g_2|)$ records in g_2 do not have a buddy in g_1 in *any* generator, so the crack size of g_2 is at least $|g_2| - min(|g_1|, |g_2|)$. According to Lemma 4.2, for some generator exactly $min(|g_1|, |g_2|)$ records in g_1 have a buddy in g_2, therefore, exactly $min(|g_1|, |g_2|)$

records in g_2 have a buddy in g_1 due to the injective buddy relationship. So, exactly $|g_2| - min(|g_1|, |g_2|)$ records in g_2 do not have a buddy in g_1. According to Theorem 4.1, these records in g_2 do not represent P. By Definition 3.3, we have

THEOREM 4.3. Suppose that a target P matches (qid_1, qid_2). Let (g_1, g_2) be in $CG(qid_1, qid_2)$. (1) g_2 has crack size c wrt P, where $c = |g_2| - min(|g_1|, |g_2|)$. (2) $C(qid_1, qid_2) = \sum c$, where $\sum$ is over (g_1, g_2) in $CG(qid_1, qid_2)$ and g_2 has the crack size c determined in (1). ∎

The condition $|g_2| > min(|g_1|, |g_2|)$ says that g_2 contains too many records even if all the records in g_1 have a buddy in g_2. In this case, some record in g_2 either comes from D_2 or has a buddy not in g_1. Such records either do not match P's timestamp (i.e., T_1) or do not originate from P's QID. It is interesting to note the condition $|g_1| - min(|g_1|, |g_2|) > 0$ for F-attack and the condition $|g_2| - min(|g_1|, |g_2|) > 0$ for B-attack are exactly opposite. More interestingly, in Section 4.4 we will show that $FA \geq k$ is violated exactly when $CA \geq k$ is violated.

EXAMPLE 4.2. Continue with Example 3.3. Alice matches qid_1=[Europe, Lawyer] and qid_2=[France, Professional]. qid_1 consists of $g_1 = \{a_1, a_2, a_3\}$ and $g_1' = \{a_4, a_5\}$. qid_2 consists of $g_2 = \{b_7, b_8\}$ and $g_2' = \{b_4, b_5, b_6\}$. $CG(qid_1, qid_2) = \{(g_1, g_2), (g_1', g_2')\}$. $|g_2| = 2$, $|g_1| = 3$, $|g_2'| = 3$ and $|g_1'| = 2$. Thus g_2 has crack size 0 and g_2' has crack size 1. ∎

4.3 B-attack

Suppose that P matches some qid_2 in R_2. Let g_2 be a group in qid_2. The crack size of g_2 is related to the number of records in g_2 that have a buddy in R_1 (thus timestamp T_1). Let G_1 denote the set of records in R_1 comparable to g_2. So G_1 contains all the records in R_1 that *can* have a buddy in g_2. Let G_2 denote the set of records in R_2 comparable to some record in G_1. The next lemma implies that all records in G_1 *can* have a buddy in $G_2 - g_2$.

LEMMA 4.3. Every record in G_2 is comparable to all records in G_1 and only those records in G_1.

PROOF. Each record r_2 in G_2 is comparable to some record r_1' in G_1 and both r_1' and r_1 are comparable to g_2, where r_1 is *any* record in G_1. It then follows from Lemma 4.1 that (r_1, r_2) are comparable. For the second part, suppose that a record r_2 in G_2 is comparable to some record r_1 in R_1. Note that r_2 is comparable to some record r_1' in G_1, which is comparable to g_2. Lemma 4.1 then implies that r_1 is comparable to g_2. By the definition of G_1, r_1 is in G_1. □

From Lemma 4.3, all records in G_1 and only those records in G_1 can have a buddy in G_2. Each record in G_1 has its buddy either in g_2 or in $G_2 - g_2$, but not in both. If $|G_1| > |G_2| - |g_2|$, the remaining $c = |G_1| - (|G_2| - |g_2|)$ records in G_1 must have their buddies in g_2, or equivalently, c records in g_2 must have their buddies in R_1 (therefore, timestamp T_1). The next theorem follows from this observation. Note that for multisets, $|G_2| - |g_2| = |G_2 - g_2|$. Also, $c \leq |g_2|$ because $|G_1| \leq |G_2|$ and all records in G_1 must have its buddy in G_2 due to the injective buddy relationship.

THEOREM 4.4. Suppose that a target P has timestamp T_2 and matches qid_2 in R_2. Let g_2 in qid_2. (1) If $|G_2| < |g_2|$, g_2 has crack size 0 wrt P. (2) If $|G_2| \geq |g_2|$, g_2 has crack size c, where $c = max(0, |G_1| - (|G_2| - |g_2|))$. (3) $B(qid_2) = \sum c$, where $\sum$ is over g_2 in qid_2 and g_2 has the crack size c determined in (1) and (2).

PROOF. (3) follows from Definition 3.6. For (1), $|G_2| < |g_2|$ implies $|G_1| = 0$ (otherwise G_2 contains at least all the records in g_2 and $|G_2| \geq |g_2|$). This means that the records in g_2 have no buddy (for any generator), thus, have timestamp T_2. In this case, g_2 has crack size 0 wrt P. We prove (2): From the discussion above, at least c (as in the theorem) records in g_2 have timestamp T_1 (for any generator). To show that c is also an upper bound, we construct a generator in which exactly c records in g_2 have timestamp T_1. In this generator, exactly c records in G_1 have their buddies in g_2 and the remaining $|G_1| - c$ records in G_1 have their buddies in $G_2 - g_2$. According to Lemma 4.3, any record in G_2 can be the buddy of any record in G_1. So we only need to show that g_2 contains at least c records and that $G_2 - g_2$ contains at least $|G_1| - c$ records. The first part follows from $|G_1| \leq |G_2|$. For the second part, if $c > 0$, $c = |G_1| - (|G_2| - |g_2|)$, so $|G_1| - c = |G_2| - |g_2|$; if $c = 0$, $|G_1| - (|G_2| - |g_2|) \leq 0$, so $|G_2| - |g_2| \geq |G_1| = |G_1| - c$. In both cases, $G_2 - g_2$ contains at least $|G_1| - c$ records. □

$|G_1| - (|G_2| - |g_2|) > 0$ occurs if $G_2 - g_2$ is so small that some records in G_1 have to find their buddies in g_2. This condition is likely to occur for a small D_2 where most records in R_2 have timestamp T_1, thus, are buddies of the records in G_1. Our experiments also confirmed this observation. In the special case of an empty D_2, all records in g_2 have timestamp T_1 and g_2 has crack size $|g_2|$. This can be derived from Theorem 4.4(2). In this special case, $|G_1| = |G_2|$ because every record in G_2 has a buddy in R_1, in particular, in G_1, according to Lemma 4.3.

EXAMPLE 4.3. Continue with Example 3.4. Alice has [UK, Lawyer] and timestamp T_2. qid_2 = [UK, Professional] has g_2 for Flu and g_2' for HIV:

$g_2 = \{b_1, b_2, b_3\}$, $G_1 = \{a_1, a_2, a_3\}$, $G_2 = \{b_1, b_2, b_3, b_7, b_8\}$
$|G_1| - (|G_2| - |g_2|) = 3 - (5 - 3) = 1$
$g_2' = \{b_9, b_{10}\}$, $G_1 = \{a_4, a_5\}$, $G_2 = \{b_4, b_5, b_6, b_9, b_{10}\}$
$|G_1| - (|G_2| - |g_2'|) = 2 - (5 - 2) = -1$

Thus g_2 has crack size 1 and g_2' has crack size 0. ∎

4.4 Equivalence of F-attack and C-attack

F-attack and C-attack are motivated under different scenarios and have a different characterization of crack size. Despite such differences, we show that these attacks are not independent of each other at all; in fact, $FA = CA$. First, we show the following lemma as a stepping stone.

LEMMA 4.4. For comparable (qid_1, qid_2), where qid_i is in R_i, $|qid_1| - F(qid_1, qid_2) = |qid_2| - C(qid_1, qid_2)$.

PROOF. $F(qid_1, qid_2) = \sum(|g_1| - min(|g_1|, |g_2|))$, following Theorem 4.2, where $\sum$ is over (g_1, g_2) in $CG(qid_1, qid_2)$. For any (g_1, g_2), if $|g_1| > |g_2|$, let (g_1^a, g_2^a) denote the pair; if $|g_1| \leq |g_2|$, let (g_1^b, g_2^b) denote the pair. Note that $\sum(|g_1| - min(|g_1|, |g_2|)) = \sum(|g_1^a| - |g_2^a|)$, and $|qid_1| = \sum |g_1^a| + \sum |g_1^b|$. So we have:

$|qid_1| - F(qid_1, qid_2)$
$= |qid_1| - \sum(|g_1| - min(|g_1|, |g_2|)) = |qid_1| - \sum(|g_1^a| - |g_2^a|)$
$= \sum |g_1^a| + \sum |g_1^b| - \sum |g_1^a| + \sum |g_2^a| = \sum |g_1^b| + \sum |g_2^a|$.

From Theorem 4.3, $C(qid_1, qid_2) = \sum(|g_2| - min(|g_1|, |g_2|))$, where $\sum$ is over (g_1, g_2) in $CG(qid_1, qid_2)$. By a similar rewriting, we have $|qid_2| - C(qid_1, qid_2) = \sum |g_2^a| + \sum |g_1^b|$. □

THEOREM 4.5. $FA = CA$.

PROOF. Refer to Definition 3.2 of FA and Definition 3.4 of CA. Let qid'_1 and qid'_2 be such that $FA = |qid'_1| - F(qid'_1, qid'_2)$. From Lemma 4.4, $FA = |qid'_2| - C(qid'_1, qid'_2)$. By the definition of CA, $CA \geq |qid'_2| - C(qid'_1, qid'_2) = FA$. Let qid^*_1 and qid^*_2 be such that $CA = |qid^*_2| - C(qid^*_1, qid^*_2)$. From Lemma 4.4, $CA = |qid^*_1| - F(qid^*_1, qid^*_2)$. By the definition of FA, $FA \geq |qid^*_1| - F(qid^*_1, qid^*_2) = CA$. Thus $FA = CA$. $\square$

At first glance, $FA = CA$ seems contradicting Theorem 4.2 and Theorem 4.3 that suggest exactly opposite conditions for F-attack and C-attack, that is, (1) $|g_1| - |g_2| > 0$ for F-attack and (2) $|g_2| - |g_1| > 0$ for C-attack. A closer look tells that there is no contradiction. Consider how condition (1) affects FA and CA. Recall that FA is the minimum $(|qid_1| - F(qid_1, qid_2))$ and CA is the minimum $(|qid_2| - C(qid_1, qid_2))$. The occurrence of condition (1) decreases FA by increasing $F(qid_1, qid_2)$. However, the occurrence of condition (1) also decreases $|qid_2|$, thus CA. So condition (1) causes both FA and CA to decrease. Similarly, condition (2) causes both FA and CA to decrease. In fact, from the proof of Lemma 4.4, both FA and CA are contributed by the same source, i.e., $\sum |g^a_2| + \sum |g^b_1|$.

5. ANONYMIZATION

We now present an algorithm for anonymizing R_2 for $D_1 \cup D_2$ to satisfy all of $FA \geq k, CA \geq k, BA \geq k$. Our approach iteratively specializes R_2 starting from the *most generalized state* of R_2. In the most generalized state, all values for each attribute $A_j \in QID$ are generalized to the top most value in the taxonomy. Each *specialization*, for some attribute in QID, replaces a parent value with an appropriate child value in every record containing the parent value. Section 5.1 shows that FA, CA, and BA are non-increasing in this specialization process. Therefore, all further specializations can be pruned once any of the above requirements is violated. Section 5.2 presents an efficient algorithm for producing such a maximally specialized R_2.

5.1 Anti-Monotonicity of BCF-Anonymity

THEOREM 5.1. Each of FA, CA and BA is non-increasing with respect to a specialization on R_2.

PROOF. From Theorem 4.5, $FA = CA$, so we show the theorem only for FA and BA. Consider a specialization on R_2 in which qid_2 is specialized into $qid^1_2, \cdots, qid^z_2$, each corresponding to a child value. Each group g_2 in qid_2 is also specialized into $g^1_2, \ldots, g^z_2$, thus, $g^j_2 \leq g_2$. Note that the specialization does not affect $|qid_1|$ for any qid_1 in R_1.

FA: Recall that FA is the minimum $(|qid_1| - F(qid_1))$ over all qid_1 in R_1. $F(qid_1) = max(F(qid_1, qid_2))$ over all qid_2 comparable to qid_1. $F(qid_1, qid_2) = \sum c$, where $\sum$ is over (g_1, g_2) in $CG(qid_1, qid_2)$ and $c = |g_1| - min(|g_1|, |g_2|)$ given by Theorem 4.2. We show that $F(qid_1)$ is non-decreasing wrt the above specialization. If qid_2 is not comparable to qid_1, qid^i_2's remain not comparable to qid_1. Assume that qid_2 is comparable to qid_1. In this case, the term $F(qid_1, qid_2)$ in $max(F(qid_1, qid_2))$ is replaced with the terms $F(qid_1, qid^i_2)$ for those qid^i_2 comparable to qid_1. For each such qid^i_2, $F(qid_1, qid_2) \leq F(qid_1, qid^i_2)$ since $|g_1| - min(|g_1|, |g_2|) \leq |g_1| - min(|g_1|, |g^i_2|)$, where (g_1, g_2) is in $CG(qid_1, qid_2)$ and (g_1, g^i_2) is in $CG(qid_1, qid^i_2)$. Therefore, $F(qid_1)$ is non-decreasing wrt the specialization.

BA: Recall that BA is the minimum $(|qid_2| - B(qid_2))$ over all qid_2 in R_2. $B(qid_2) = \sum c$, where $\sum$ is over the groups g_2 in qid_2 and c is the crack size of g_2. Unlike FA, the specialization on R_2 decreases both $|qid_2|$ and $B(qid_2)$. We show that the decrease of $|qid_2|$ is *at least as much as* the decrease of $B(qid_2)$, which implies that $|qid^i_2| - B(qid^i_2) \leq |qid_2| - B(qid_2)$. Consider obtaining the specialized equivalence class qid^i_2 from qid_2 by discarding all groups g^j_2 of qid^j_2, $j \neq i$. Discarding g^j_2 decreases $|qid_2|$ by $|g^j_2|$ and decreases $B(qid_2)$ by at most c^j, where c^j is the crack size of g^j_2. Note that $c^j \leq |g^j_2|$. Therefore, the decrease of $|qid_2|$ is at least as much as that of $B(qid_2)$. $\square$

COROLLARY 5.1. For a given requirement on BCF-anonymity, there exists a generalized R_2 that satisfies the requirement if and only if the most generalized R_2 does. ∎

5.2 Algorithm

Finding an optimal BCF-anonymized R_2 is NP-hard. Our approach *BCF-anonymizer*, summarized in Algorithm 1, aims at producing a maximally specialized (suboptimal) BCF-anonymized R_2 which any further specialization leads to a violation. It starts with the most generalized R_2. At any time, R_2 contains the generalized records of $D_1 \cup D_2$ and $Cut2_j$ gives the generalization cut for $A_j \in QID$. Each equivalence class qid_2 in R_2 is associated with a set of groups g_2 with stored $|g_2|$. Each group g_2 is associated with the set of raw records in $D_1 \cup D_2$ generalized to the group. R_1 is represented similarly with $|qid_1|$ and $|g_1|$ stored, except that no raw record is kept for g_1. $Cut1_j$ contains the generalization cut $A_j \in QID$ in R_1. $Cut1_j$ never change once created.

Initially, $\cup Cut2_j$ and the candidate list contain the most general value ANY_j for every $A_j \in QID$ (Lines 1-3). In each iteration, we examine the first valid candidate specialization ranked by a criterion $Score$. If the candidate w is *valid*, that is, not violating the BCF-anonymity after its specialization, we specialize w on R_2 (Lines 6-10); otherwise, we remove w from the candidate list (Line 12). This iteration is repeated until there is no more candidate. From Theorem 5.1, the returned R_2 is maximal (suboptimal). $Score$ ranks the candidates by their "information worth." We employ the discernibility cost [12] which charges a penalty to each record for being indistinguishable from other records. For each record

Algorithm 1 BCF-anonymizer
Input: R_1, $R_2 = D_1 \cup D_2$, k, taxonomy for each $A_j \in QID$.
Output: a BCF-anonymized R_2.

```
1:  generalize every value for $A_j \in QID$ in $R_2$ to $ANY_j$;
2:  let candidate list = $\cup Cut2_j$ containing all $ANY_j$;
3:  sort candidate list by $Score$ in descending order;
4:  while the candidate list is not empty do
5:      if the first candidate $w$ in candidate list is valid then
6:          specialize $w$ into $w_1, \cdots, w_z$ in $R_2$;
7:          compute $Score$ for all $w_i$;
8:          remove $w$ from $\cup Cut2_j$ and the candidate list;
9:          add $w_1, \cdots, w_z$ to $\cup Cut2_j$ and the candidate list;
10:         sort the candidate list by $Score$ in descending order;
11:     else
12:         remove $w$ from the candidate list;
13:     end if
14: end while
15: output $R_2$ and $\cup Cut2_j$;
```

in an equivalence class qid_2, this penalty is $|qid_2|$. To minimize the discernibility cost, we choose the specialization w that maximizes $Score(w)$, where $Score(w) = \sum_{qid_w} |qid_w|^2$ over all qid_w containing w.

In general, Lines 5-7 require scanning all pairs (qid_1, qid_2) and all records in R_2, which is highly inefficient for a large data set. We present an incremental computation of FA, CA, and BA that examines only comparable pairs (qid_1, qid_2) and raw records in R_2 that are *involved* in the current specialization. Let $\{w_1, \cdots, w_z\}$ be the set of child values of the winning candidate w on Line 6. Let qid_w be a qid_2 containing w and let $qid_{w_1}, \ldots, qid_{w_z}$ be the new qids resulting from specializing qid_w. Let g_w be a group in qid_w and g_{w_i} be a group in qid_{w_i}.

Line 6: To perform the specialization of w efficiently, for each value v in $\cup Cut2_j$, we create $Link[v]$ to link up all qid_2 containing v. $Link[w]$ provides a direct access to all equivalence classes qid_w and associated records to be specialized. After the specialization, we create new $Link[w_i]$ to link up new qid_{w_i} containing w_i. In addition, we add qid_{w_i} to every existing $Link[v]$ to which qid_w was previously linked, except for $Link[w]$. The overhead for maintaining these links is negligible. [4] shows that the time complexity of this specialization procedure is linear in $|R_2|$.

Line 7: In the same scan of $Link[w_i]$, $Score(w_i)$ is computed by $\sum |qid_{w_i}|^2$ for all qid_{w_i} on $Link[w_i]$.

Line 5: To check the validity of w, we introduce a data structure, called $GTree1$, to index all comparable qid_1 in R_1 for a given qid_2 in R_2. $GTree1$ has one level per attribute in QID followed by the leaf level for the sensitive attribute. Each root-to-leaf path in $GTree1$ represents a group g_1 in R_1. To find all qid_1 in $GTree1$ that are comparable to qid_2, we traverse $GTree1$ in a depth-first manner. At the level for an attribute $A \in QID$, if $qid_2[A]$ and $qid_1[A]$ are not on the same path in the taxonomy of A, the entire subtree below $qid_1[A]$ is pruned. On reaching a node on the level $|QID|$, the qid_1 at the node is comparable to qid_2. $GTree1$ is static. Below, we update FA and BA using $GTree1$.

Procedure 2 Updating FA

```
1: for all new $qid_{w_i}$ on $Link(w_i)$ do
2:     for all comparable $qid_1$ do
3:         let $F(qid_1) = max(F(qid_1), F(qid_1, qid_{w_i}))$;
4:         let $FA = min(FA, |qid_1| - F(qid_1))$;
5:     end for
6: end for
```

Updating FA: FA is the minimum $(|qid_1| - F(qid_1))$ over all qid_1 in R_1, where $F(qid_1) = max(F(qid_1, qid_2))$ over all qid_2 comparable to qid_1. $F(qid_1, qid_2)$ is a function of $|g_1|$ and $|g_2|$ for (g_1, g_2) in $CG(qid_1, qid_2)$. We maintain $|g_2|$, FA, and $F(qid_1)$ on performing each specialization. For each qid_{w_i} on $Link[w_i]$, find all comparable qid_1 (using $GTree1$) and update $F(qid_1)$ and FA as shown in Procedure 2. The update on Line 3 exploits the property that $F(qid_1)$ is non-decreasing wrt a specialization on R_2 (Theorem 5.1). The computation is proportional to the number of comparable (qid_1, qid_{w_i}) for all new qid_{w_i} added in the current iteration.

Updating BA: BA is the minimum $(|qid_2| - B(qid_2))$ over all qid_2 in R_2, where $B(qid_2)$ is a function of $|G_1(g_2)|$, $|G_2(g_2)|$ and $|g_2|$ for the groups g_2 in qid_2. $G_1(g_2)$ denotes the set of groups in R_1 comparable to g_2, $G_2(g_2)$ denotes the set of groups in R_2 comparable to any group in $G_1(g_2)$. We maintain $|G_1(g_2)|$, $|G_2(g_2)|$ and $|g_2|$ on specializing qid_2.

Procedure 3 Updating BA (Case 1)

```
1: for all new $qid_{w_i}$ on $Link[w_i]$ do
2:     let $|G_1(g_{w_i})| = 0$; let $|G_2(g_{w_i})| = 0$;
3:     let $compqid1$ be the set of $qid_1$ comparable to $qid_{w_i}$;
4:     for all $qid_1$ in $compqid1$ do
5:         let $|G_1(g_{w_i})| = |G_1(g_{w_i})| + |g_1|$, where $(g_1, g_{w_i}) \in$
           $CG(qid_1, qid_{w_i})$;
6:     end for
7:     for all new $qid'_{w_i}$ on $Link[w_i]$ do
8:         if $qid'_{w_i}$ is comparable to some $qid_1$ in $compqid1$ then
9:             let $|G_2(g_{w_i})| = |G_2(g_{w_i})| + |g'_{w_i}|$, where $g'_{w_i}$ of $qid'_{w_i}$
               has the same sensitive value as $g_{w_i}$ of $qid_{w_i}$;
10:        end if
11:    end for
12:    let $BA = min(BA, |qid_{w_i}| - B(qid_{w_i}))$;
13: end for
```

We need to compute $B(qid_{w_i})$ *only* for new qid_{w_i} because $B(qid_2)$ of unspecialized qid_2 remains intact. To compute $B(qid_{w_i})$, first we compute $|G_1(g_{w_i})|$, $|G_2(g_{w_i})|$ and $|g_{w_i}|$ for the new g_{w_i} in qid_{w_i}. Let qid_1 in R_1 be comparable to qid_w. Assume w is from attribute A_j. There are two cases.

Case 1: w is above the cut $Cut1_j$. In this case, only qid_{w_i} with w_i being equal to or more general than $qid_1[A_j]$ is comparable to qid_1. This means that, to compute $G_2(g_{w_i})$, we only need to examine g'_{w_i} of those qid'_{w_i} on $Link[w_i]$. Based on this observation, Procedure 3 updates $B(qid_{w_i})$ and BA. Lines 3-6 computes $|G_1(g_{w_i})|$ and Lines 8-12 computes $|G_2(g_{w_i})|$. Each assignment involving a group applies to all groups in an equivalence class.

Case 2: w is on or below the cut $Cut1_j$. In this case, every new qid_{w_i} remains comparable to qid_1. Therefore, for g_{w_i} of qid_{w_i}, $|G_1(g_{w_i})| = |G_1(g_w)|$ and $|G_2(g_{w_i})| = |G_2(g_w)|$, where g_w is the corresponding group of qid_w. Note that $|G_1(g_w)|$ and $|G_2(g_w)|$ were stored with g_w.

6. EMPIRICAL STUDY

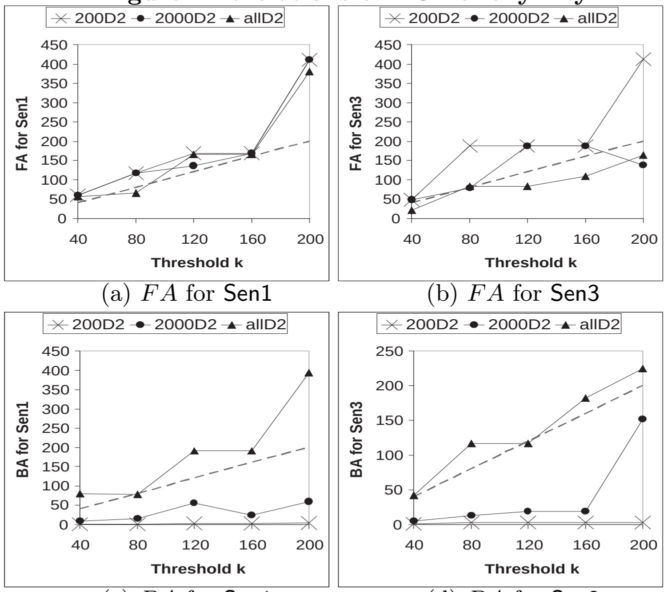
We studied the threat of correspondence attacks and the usefulness of the BCF-anonymized R_2. We employed a publicly available census data set, *Adult*, previously used in [2, 4, 6, 8, 14, 15]. After removing all records with missing values, there were 30,162 and 15,060 records in the training and testing sets on 8 categorical attributes. Refer to [4] for the properties and taxonomy of the attributes. We specified D_1 to contain all records in the testing set, and considered the following three cases of D_2 at timestamp T_2:

- $200D2$: D_2 contains the first 200 records in the training set, modelling a "small" set of new records at T_2, typically true if the new data is published in a timely manner.
- $2000D2$: D_2 contains the first 2000 records in the training set, modelling a "medium" set of new records at T_2.
- $allD2$: D_2 contains all 30,162 records in the training set, modelling a "large" set of new records at T_2.

We specified two choices of sensitive attributes. In Sen1, the sensitive attribute is chosen to be the attribute having the most number of distinct values, which is *Native-country*, and QID contains the remaining 7 attributes. In Sen3, the sensitive attribute is chosen to be the three attributes having the most number of distinct values, which is {*Native-country*, *Education*, *Occupation*}, and QID contains the remaining 5 attributes. Sen3 has a more restrictive buddy relationship than Sen1.

Two sets of experiments were conducted. In the first set,

Figure 1: Violations of BCF-anonymity



(a) FA for Sen1 (b) FA for Sen3

(c) BA for Sen1 (d) BA for Sen3

we studied the violation of BCF-anonymity when both R_1 and R_2 are k-anonymized individually as proposed in [5]. In the second set, we anonymized R_2 by the BCF-anonymizer and evaluated its quality. All experiments were conducted on Pentium IV 2.4GHz PC with 512MB of RAM. The BCF-anonymizer took less than 7 seconds, including disk I/O operations. This shows that the algorithm is highly effective.

6.1 Detecting Violations

For each pair of (D_1, D_2) described above, we generalized D_1 and $D_1 \cup D_2$ into k-anonymized R_1 and k-anonymized R_2 separately, by modifying the algorithm in Section 5 to enforce the k-anonymity on a single table without concerning correspondence attacks. We then measured FA, CA and BA on the generalized R_1 and R_2. Since $FA = CA$ (Theorem 4.5), which was also confirmed by our experiments, we present the results for F-attack and B-attack.

F-attack: Figures 1a-1b depict FA for $40 \leq k \leq 200$ for Sen1 and Sen3 in $200D2$, $2000D2$, and $allD2$. k is the threshold for k-anonymization. The dash line represents the boundary for $FA = k$. A data point below the boundary indicates $FA < k$, i.e., a violation of F-anonymity by k-anonymized R_1 and R_2.

While FA *generally* increases as k increases, there is no guarantee that $FA \geq k$. Figure 1b shows that 4 out of the 5 test cases have $FA < k$ for Sen3 and $allD2$. The most severe case occurs at $k = 40$ and $FA = 21$, where the anonymity of some individuals has been decreased by 48% due to F-attack. Staying above the dash line does not mean no cracked records; rather, it only means that the F-anonymity stays above k. FA does not increase monotonically as k increases. There are two reasons. First, with a larger k, R_1 was less specialized, which led to a larger crack size in R_1. Second, with a larger k, some specialization on R_2 valid for a smaller k became invalid and a different sequence of specializations on R_2 was exploited.

A larger size $|D_2|$ has mixed effects on the crack size $|g_1| - min(|g_1|, |g_2|)$: having more records at T_2 means larger $|g_2|$, but also means less generalization in R_2, thus, smaller $|g_2|$. For the more restrictive buddy relationship of Sen3, violations occur more frequently in the case of $allD2$ where the decrease of $|g_2|$ becomes the dominant factor due to less generalization in R_2. Comparing Figures 1a-1b, Sen3 suffers more violations than Sen1. As the sensitive attribute gets more restrictive for Sen3, both $|g_1|$ and $|g_2|$ become smaller. For the larger number of new records in $allD2$, however, the decrease of $|g_2|$ dominates, therefore causes more violations.

B-attack: Figures 1c-1d depict BA for Sen1 and Sen3. Unlike F-attack, with fewer new records in D_2 (i.e., $200D2$ and $2000D2$), violations $BA < k$ occur more frequently and severely since many records in an equivalence class have timestamp T_1, which are deemed unrelated to the target having timestamp T_2. The most severe case occurs at $k = 80$ for Sen1 in $200D2$ with $BA = 1$. In other words, some target was *uniquely* identified since all other matching records in R_2 were correctly identified as having timestamp T_1. Another difference from F-attack is that violations $BA < k$ occur for both Sen1 and Sen3, and are more frequent and more severe for $200D2$ and $2000D2$ than for $allD2$.

This experiment suggested that a small or medium D_2 is highly vulnerable to B-attack. As the increment D_2 is typically small, the threat of B-attack is very real. A large D_2 can be vulnerable to F-attack due to the reduced generalization in R_2. This is especially so for a restrictive sensitive attribute. As a result, for neither small nor large D_2 is the k-anonymity safe from correspondence attacks.

6.2 Preventing Violations

This experiment evaluates the quality of BCF-anonymized R_2. The quality is measured by the discernibility cost [12] for R_2, which is defined as the square sum of the sizes of all equivalence classes in R_2, normalized by $|R_2|^2$. The normalized cost has the range from 0 to 1, with 0 being the best and 1 being the worst. For each run in the detection experiment, if there is a violation, we compare the discernibility cost of the following three alternatives.

- *BCF-anonymized* R_2: R_2 is generalized by our BCF-anonymizer to satisfy *all* of $FA \geq k$, $CA \geq k$, $BA \geq k$.
- *k-anonymized* R_2: This is the k-anonymized R_2 in the detection experiment and in [5]. This is not safe from correspondence attacks.
- *k-anonymized* D_2: The records in D_2 are anonymized separately from those in D_1. This alternative is safe because correspondence attacks cannot be applied to data sets representing disjoint sets of individuals.

We evaluate the data quality of a BCF-anonymized R_2 as follows. (1) We measure the penalty to achieve additional protection against all correspondence attacks, relative to the unsafe k-anonymized R_2. (2) We measure the benefit to combine D_1 and D_2 into one release R_2 for anonymization, relative to the safe k-anonymized D_2. Figure 2 depicts the discernibility cost of the three alternatives. Note that Sen3 has a cost higher than Sen1 since more attributes are exactly preserved through the sensitive attribute and more generalization on the remaining QID attributes is required to achieve anonymity.

BCF-anonymized R_2 *vs* k*-anonymized* D_2. For a smaller D_2, i.e., $200D2$ and $2000D2$, BCF-anonymized R_2 has a cost lower than k-anonymized D_2. For $200D2$, the average cost (over the five test cases) of BCF-anonymized R_2 is 66% and 32% lower than k-anonymized D_2 for Sen1 and Sen3, shown in Figures 2a-2b. This significant cost reduction benefits from anonymizing a large $D_1 \cup D_2$. When D_2 contains a relatively large number of new records, i.e., *all*$D2$ in Figures 2e-2f, the two alternatives have a similar cost since they tend to focus on new records. From this point of view, a separate anonymization of D_2 makes sense if the new data is abundant. Typically, however, the increment D_2 is small.

BCF-anonymized R_2 vs k-anonymized R_2. BCF-anonymized R_2 generally has a higher cost than k-anonymized R_2. The cost increases mainly for $200D2$ and $2000D2$, as in Figures 2a-2d. A cross-examination of Figures 1c-1d reveals that for $200D2$ and $2000D2$ there is a very severe violation of $BA \geq k$ across all k. The violation was so severe that some target individuals were uniquely identified by the B-attack. To prevent such severe violations, for example, the average increase of cost is 25% in Figure 2d. k-anonymized R_2 is completely unprotected although it has a lower cost.

In Figure 2c, BCF-anonymized R_2 has a cost lower than k-anonymized R_2 at $k = 40$, though the requirement is more restrictive. In this case, some top ranked specializations used for k-anonymized R_2 became invalid for BCF-anonymizer due to the additional requirement $BA \geq k$, and some lower ranked specializations were selected, which actually resulted in a smaller overall cost. This is possible since the solutions are suboptimal. For a similar reason, a larger k could possibly produce a lower cost than a smaller k.

7. EXTENSIONS

7.1 Beyond Two Releases

We extend the two-release case to the general case involving more than two releases. Consider the raw data $D_1, \cdots, D_n$ collected at timestamp $T_1, \cdots, T_n$. Let R_i denote the release for $D_1 \cup \cdots \cup D_i$, $1 \leq i \leq n$. All records in R_i have the special timestamp, denoted by T_i^*, that matches *any* timestamp from $T_1, \cdots, T_i$. The correspondence knowledge now has the form that every record in R_i (except the last one) has a corresponding record in all releases R_j such that $j > i$. The notion of "generators" can take this into account. Given more releases, the attacker can conduct two additional types of correspondence attacks described below.

Optimal micro attacks: The general idea is to choose the "best" background release, yielding the largest possible crack size, *individually* to crack each group. Consider the F-attack on R_i as an example. The attacker now can choose any R_j with $j > i$ as the background release because the correspondence knowledge holds for (R_i, R_j). Suppose that the target P has timestamp T_i and matches $(qid_i, \cdots, qid_n)$ and that $(g_i, \cdots, g_n)$ are the corresponding groups that have the same sensitive value. Let c_{ij} be the crack size of a group g_i computed by Theorem 4.2 wrt (g_i, g_j) of (R_i, R_j) instead of (g_1, g_2) of (R_1, R_2). To the attacker, the "optimal" crack size of g_i is $max_j\{c_{ij}\}$ over all $j > i$. The number of cracked records in an equivalence class qid_i wrt P, i.e., $F(P, qid_i, \cdots, qid_n)$, is given by $\sum_{g_i} c_i$, where g_i is a group in qid_i and $c_i = max_j\{c_{ij}\}$ is the optimal crack size of g_i.

Essentially, the threat of the F-attack on g_i is determined by the largest crack size c_{ij} contributed by a collection of "micro attacks" at the group level that each involves (the groups of) two releases at a time, i.e., (g_i, g_j). For such micro attacks, the crack size c_{ij} can be computed by our method for two releases. Our insight is that it suffices to consider only micro attacks involving two releases. Suppose that a micro attack on g_1 employs the groups g_2 and g_3 from

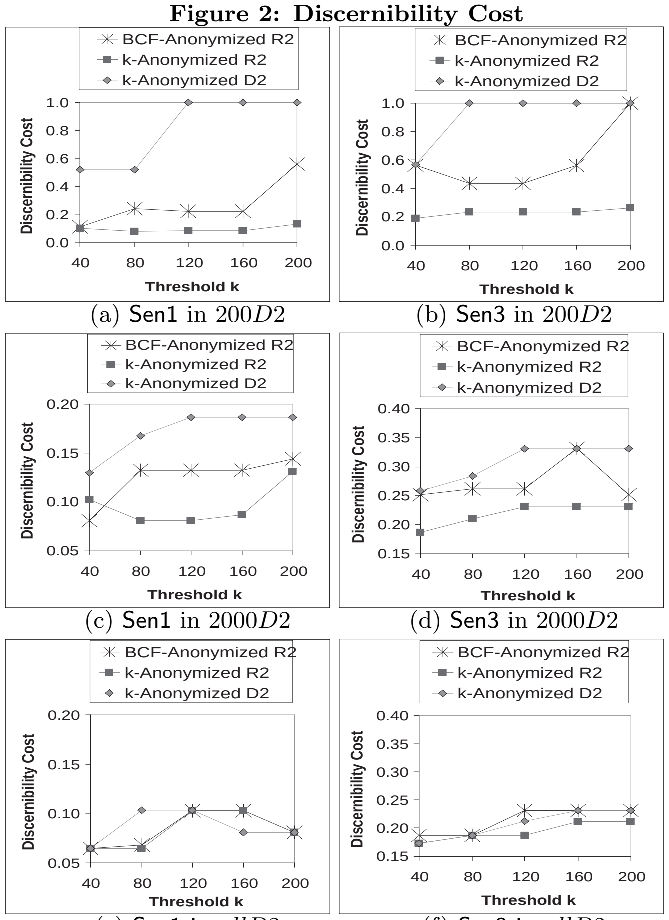


Figure 2: Discernibility Cost

(a) Sen1 in $200D2$ (b) Sen3 in $200D2$

(c) Sen1 in $2000D2$ (d) Sen3 in $2000D2$

(e) Sen1 in *all*$D2$ (f) Sen3 in *all*$D2$

two background releases R_2 and R_3. Similar to Theorem 4.2, we can show that the crack size in this case is equal to $c = |g_1| - min(|g_1|, |g_2|, |g_3|)$. But c is exactly the above optimal crack size of g_1 because $c = max(c_{12}, c_{13})$, where $c_{12} = |g_1| - min(|g_1|, |g_2|)$ is the crack size produced by the micro attack using (g_1, g_2) and $c_{13} = |g_1| - min(|g_1|, |g_3|)$ is the crack size produced by the micro attack using (g_1, g_3).

The micro attacks for the three types of attacks can be represented as follows. Note that the crack size for these micro attacks can be directly computed by the methods in Section 4. Let g_i be a group in R_i and g_j be a group in R_j.

- F-attack(g_i, g_j), $j > i$: The target P has timestamp T_i and the attacker tries to crack the records in g_i using g_j of the background release R_j. In this case, a cracked record does not originate from P's QID.
- C-attack(g_i, g_j), $j > i$: The target P has timestamp T_i and the attacker tries to crack the records in g_j using g_i of the background release R_i. A cracked record either does not have timestamp T_i^* or does not originate from P's QID. Note that if a record in g_j does not have timestamp T_i^*, it has a timestamp from $T_{i+1}, \cdots, T_j$, therefore, does not match P's timestamp.
- B-attack(g_i, g_j), $j > i$: The target P has timestamp T_j and the attacker tries to crack the records in g_j using g_i of the background release R_i. Note that a cracked record has timestamp T_i^* (because of coming from g_i), thus, does not match P's timestamp.

Composition of micro attacks: Another type of attack is to "compose" multiple micro attacks together (apply *one* *after another*) in order to increase the crack size of a group. Composition is possible *only if* all the micro attacks in the composition assume the same timestamp for the target and the correspondence knowledge required for the next attack holds after applying previous attacks. There are two and only two possible cases of composition.

Case 1: Apply B-attack(g_i, g_j) followed by F-attack(g_j, g_l), $l > j > i$. Both attacks assume the target P at timestamp T_j and crack records in g_j wrt P. First, apply B-attack(g_i, g_j) to crack g_j. This excludes some records at timestamp T_i^*. The crack size, x_B, is computed by Theorem 4.4. Let g'_j and R'_j denote the reduced g_j and R_j without cracked records. Note that the following correspondence knowledge holds: every record in R'_j has a buddy in R_l since R'_j is a subset of R_j. Hence, F-attack(g'_j, g_l) can be applied to crack g'_j, which excludes some records from g'_j that do not originate from P's QID. The crack size, x_F, is given by Theorem 4.2. Overall, the crack size of g_j is $x_B + x_F$. Note that if F-attack(g_j, g_l) is applied first to crack g_j, B-attack(g_i, g'_j) cannot be applied since a record in R_i may not have a buddy in R'_j.

Case 2: Apply B-attack(g_i, g_j) followed by C-attack(g_j, g_l), $l > j > i$. Both attacks assume a target P at timestamp T_j. First, apply B-attack(g_i, g_j) to crack g_j, which excludes some records that do not have timestamp T_j (thus do not represent P). Let g'_j and R'_j be the reduced g_j and R_j. Since every record in R'_j has a buddy in R_l, C-attack(g'_j, g_l) can be applied to crack g_l. From Theorem 4.3, the crack size of g_l by this C-attack is $|g_l| - min(|g'_j|, |g_l|)$, which is larger than $|g_l| - min(|g_j|, |g_l|)$ by applying C-attack(g_j, g_l) alone.

All other cases, however, cannot be composed. For example, F-attack(g_i, g_j) and C-attack(g_j, g_l) cannot be applied in sequence because the former assumes a target P at timestamp T_i whereas the latter assumes P at timestamp T_j.

We extend the anonymization algorithm as follows. (1) The notion of BCF-anonymity should be defined based on the optimal crack size of a group wrt micro attacks as well as composed attacks on the group. (2) Each time we anonymize the next release R_n for $D_1 \cup \cdots \cup D_n$, we assume that $R_1, \cdots, R_{n-1}$ satisfy BCF-anonymity. Hence, the anonymization of R_n only needs to ensure that BCF-anonymity is not violated by any attack that *involves* R_n. (3) The anti-monotonicity of BCF-anonymity, in the spirit of Theorem 5.1, remains valid in this general case. These observations are crucial for maintaining the efficiency.

As the number of releases increases, the constraints imposed on the next release R_n become increasingly restrictive. However, this does not necessarily require more distortion because the new records D_n may help reduce the need of distortion. In case that the distortion becomes too severe, the data holder may consider starting a new chain of releases without including previously published records.

7.2 Beyond Anonymity

The proposed approach can be extended to incorporate with the requirement of entropy ℓ-diversity and (c,ℓ)-diversity in [8], confidence bounding [15], and (α,k)-anonymity [16]. First, we modify these measures to take into account the exclusion of cracked records. In this case, the crack size of each group gives all the information needed to exclude sensitive values from an equivalence class. To extend the anonymization algorithm, we argue that anonymity is a *necessary* privacy property because identifying the exact record of an individual from a small set of records is too easy. Thus, BCF-anonymity is required even if other privacy requirements are desired. Under this assumption, we can still apply the proposed approach to prune unpromising specializations based on the anti-monotonicity of BCF-anonymity.

8. CONCLUSION

We considered the anonymity problem for a scenario where the data are continuously collected and published. Each release contains the new data as well as previously collected data. Even if each release is k-anonymized, the anonymity of an individual can be compromised by cross-examining multiple releases. We formalized this notion of attacks and presented a detection method and an anonymization algorithm to prevent such attacks. Finally, we showed that both the detection and the anonymization methods are extendable to deal with multiple releases and other privacy requirements.

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