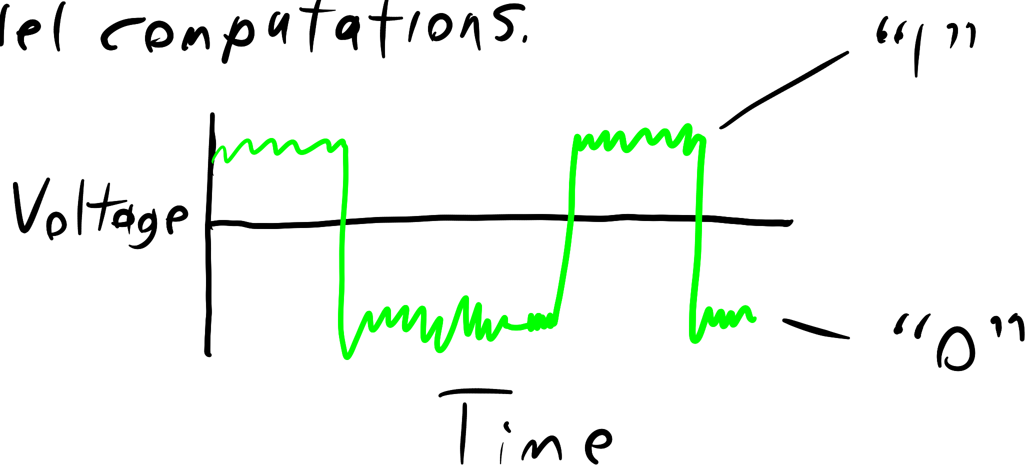
CMPT 476: Introduction to Quantum Algorithms!

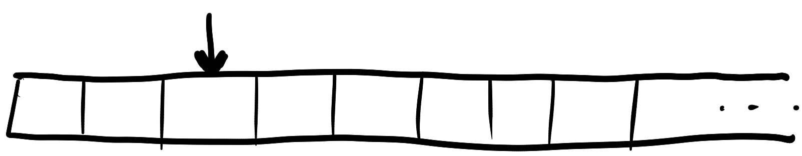
Lecture 1: What the heck is QC?

To understand what quantum computing is, we first need to understand classical computing!

Computation is a physical process of calculation (sidebar: we could instead say well defined, but how do we define well-defined?)
We use abstractions to describe and model computations.



A common abstraction is a Turing machine



(Complexity)

We often want to know how difficult it is to compute something. Without (yet) saying what we mean by that, the hope is that our abstraction maps easy (physical) problems to easy abstract problems

Physical		Abstract
Easy	$\longleftrightarrow$	Easy
Hard	$\longleftrightarrow$	Hard

(Extended Church-Turing thesis)

Any reasonable (physical) model of computation can be efficiently (easy → easy) simulated by a probabilistic Turing machine

In other words, we can forget about physics because we can't do anything* physically efficiently that we can't do efficiently with a digital computer.

* → anything programmable in the input → output sense

(Feynman, 1982)

A quantum mechanical process can NOT be simulated efficiently by any classical model of computation.

$\Downarrow$

Violation of ECTT!

... But can we use quantum mechanical systems to perform computations we care about? Yes!!!

- (Shor 1994) Integer factorization
- (Lloyd 1996) Hamiltonian simulation
- (Grover 1996) Unstructured search
- (HHL 2008) Sparse linear systems

And more in this course!

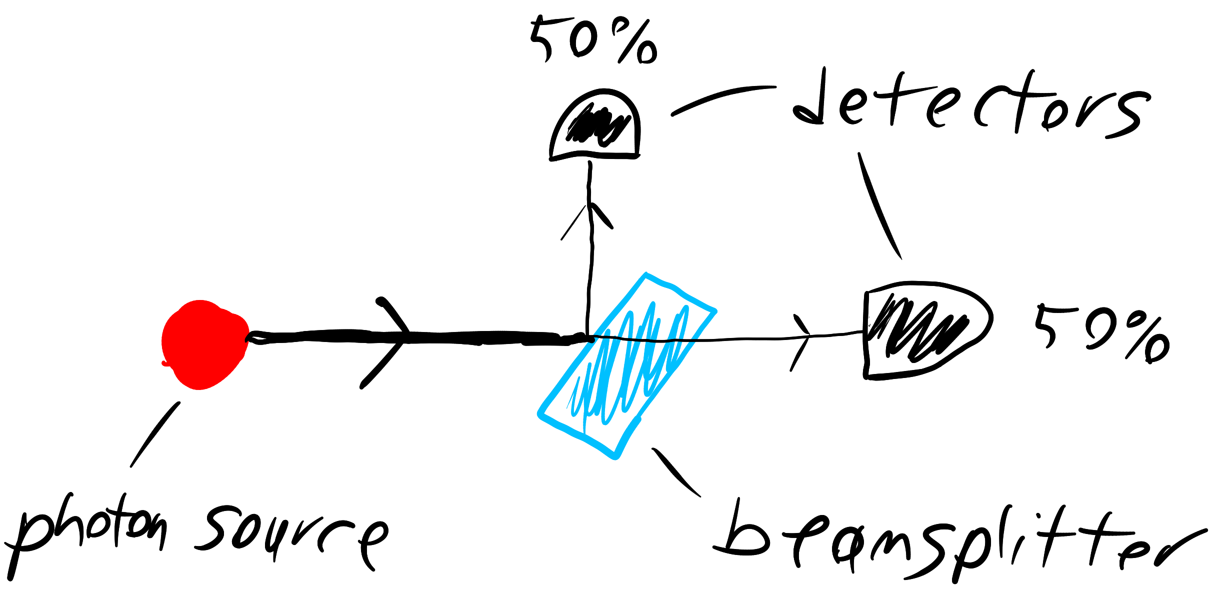
(A preview of Quantum computation)

Shopping list for QC:

1. Two (2) distinct and measurable... physical states $|0\rangle$ & $|1\rangle$
2. The ability to be in a superposition of them
3. The ability to generate interference

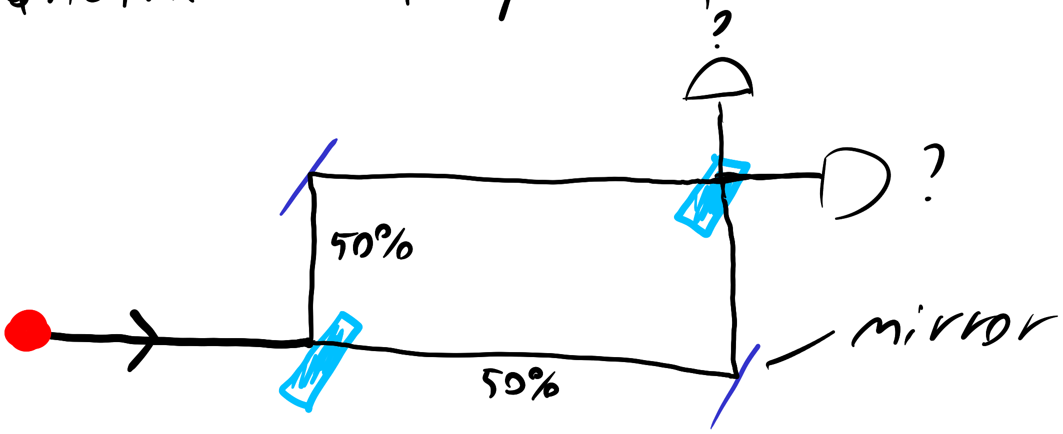
(Interferometers)

A classic example of all 3 ingredients is an interferometer. Given a photon source (light) (e.g. laser), a beam splitter reflects photons with 50% probability.



So, if we send a single photon through this set up, it should be detected in either location with 50% probability.

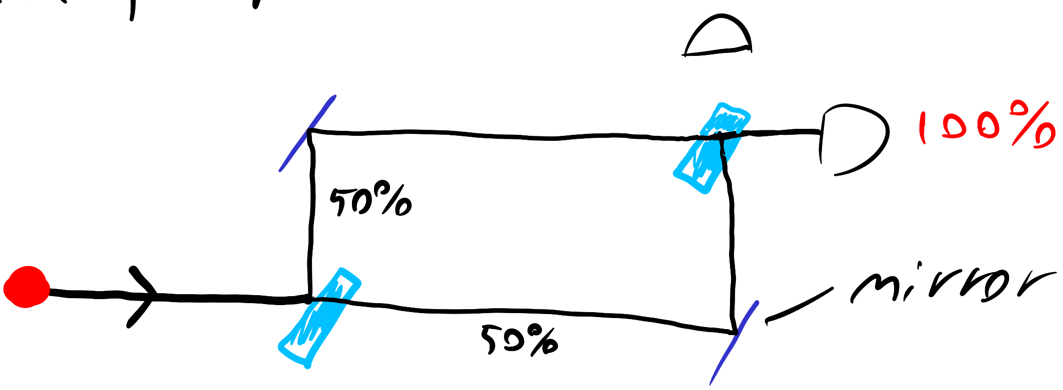
If we redirect the two beams of light back into another beamsplitter, what would happen?



Classically, we should detect at either location with 50% probability:

- 2 ways to get to the top detector, ↑ then ↑ or → then ↑
- each path has $0.5 \cdot 0.5 = 0.25$ probability
- total is $0.25 + 0.25 = 0.5$

However, in practice we find



The reason is the photon took all paths at the same time (superposition) and the paths leading to the upper detector cancelled out (interference)

(Linear algebraic model)

Mathematically, we model a superposition by a linear combination of vectors.

- $|0\rangle = \begin{pmatrix}1\\0\end{pmatrix}$ is the initial/transmited path
- $|1\rangle = \begin{pmatrix}0\\1\end{pmatrix}$ is the reflected path
- A photon is in the state

  complex numbers

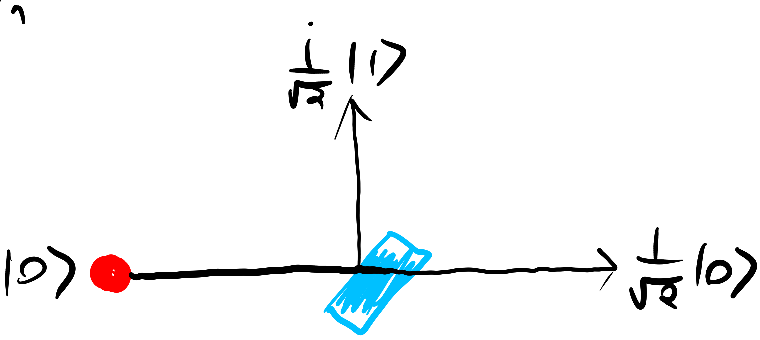
  $$\alpha|0\rangle + \beta|1\rangle, \quad \alpha, \beta \in \mathbb{C}$$

- If we were to measure the state by placing photon detectors along either path, we would find the photon in state $|0\rangle$ with probability $|\alpha|^2$ (similar for $|1\rangle$)

The beam splitter is modeled by applying the matrix $\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}$ to the state vector:

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}|0\rangle = \frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$$

Pictorally,



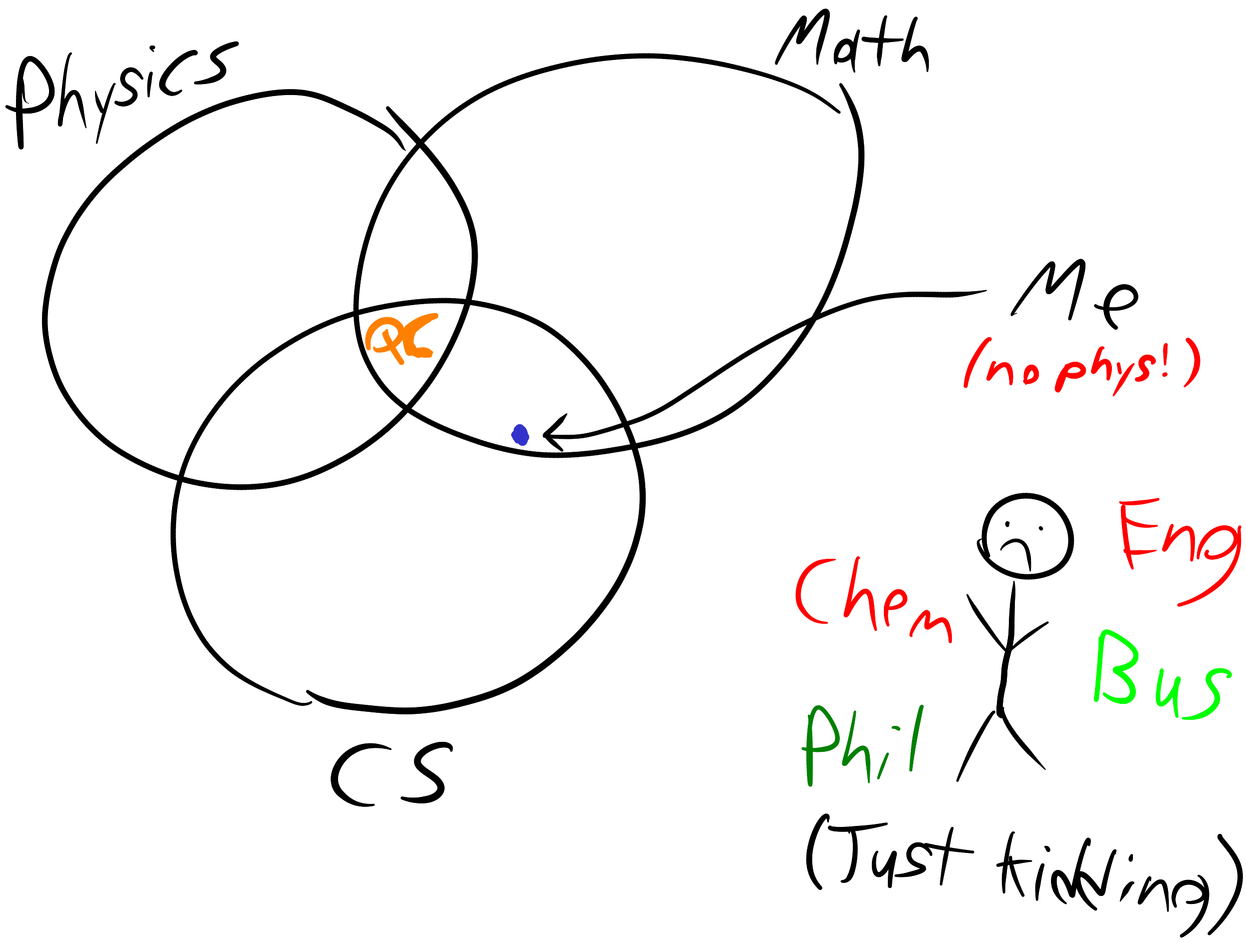
When we apply the second beam splitter, the photon is in the state $\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$, so the resulting state is

$$\frac{1}{\sqrt{2}}\begin{bmatrix}1 & i\\ i & 1\end{bmatrix}\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix} = \frac{1}{2}\begin{bmatrix}1+i^2\\ i+i\end{bmatrix} = \begin{bmatrix}0\\i\end{bmatrix} = i|1\rangle$$

Hence we detect the photon along the transmitted path $|1\rangle$

(Does this actually correspond to reality?)

A better question is does it matter? As long as our abstract model can predict the outcome of the physical process, we can put our heads in the sand and forget about the physics



(Housekeeping)

Website: (check often!)

cs.sfu.ca/~meamy/Teaching/s24/cmpt476

Evaluation:

50% assignments (approx 6)

15% mid-term exam

35% final exam

TAs:

Lucas Stinchcombe (CS)

Ming Yin (Math)

Resources:

See website!