CMPT 476 Lecture 13

Quantum computing

Up to this point we've looked in general at quantum mechanics, quantum information, and a few example protocols that we can implement with a few qubits. Now we're ready to shift gears and develop a general notion of quantum computation and see how it compares to classical computation.

First let's review computational complexity and the circuit model so that we'll be able to make a meaningful comparison.

(Computational complexity)

Our original motivation for looking at quantum computation was that simulating quantum dynamics on a classical computer was not "efficient."
We can formally state what we mean by this using order notation and complexity theory.

We say an algorithm (e.g. a python program) with input x runs in time

$$O(f(n)), \quad f: \mathbb{N} \to \mathbb{R}$$

if there exists $n_0 \in \mathbb{N}$, $c \in \mathbb{R}$ such that whenever x has length $n \geq n_0$, the runtime of the algorithm is at most

$$c \cdot f(n)$$

We say that an algorithm runs in polynomial time if

$$f(n) \approx n^k, \quad k \in \mathbb{R}$$

and exponential time if

$$f(n) \approx k^n, \quad k \in \mathbb{R}$$

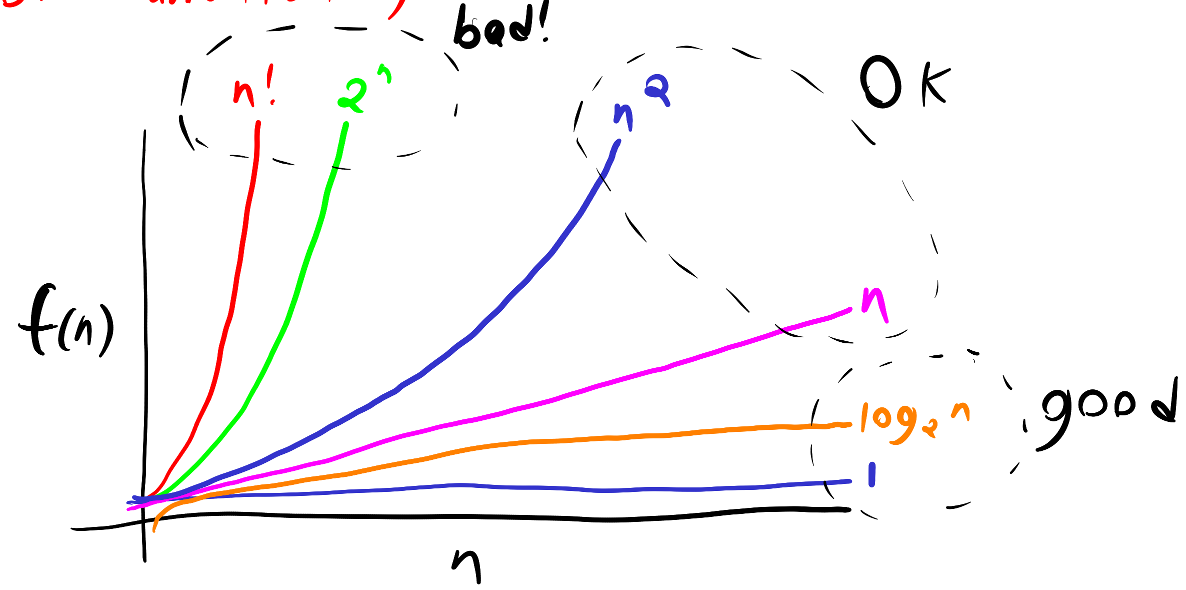
The key thing to note is that the runtime of an algorithm running in exponential time grows much, much faster than one running in polynomial time.

Ex.
Let algorithm A run in time $O(n^2)$ and algorithm B run in time $O(2^n)$. Suppose the input has length 100. Then, if the runtime is in seconds, Alg. A would take

$$100^2 \approx 3 \text{ hours.}$$

Algorithm B on the other hand would take 2^{100} seconds, which is older than the age of the universe!

(Growth of functions)



(Complexity classes)

In theoretical computer science, we classify problems (e.g. sorting an array) in terms of their complexity.

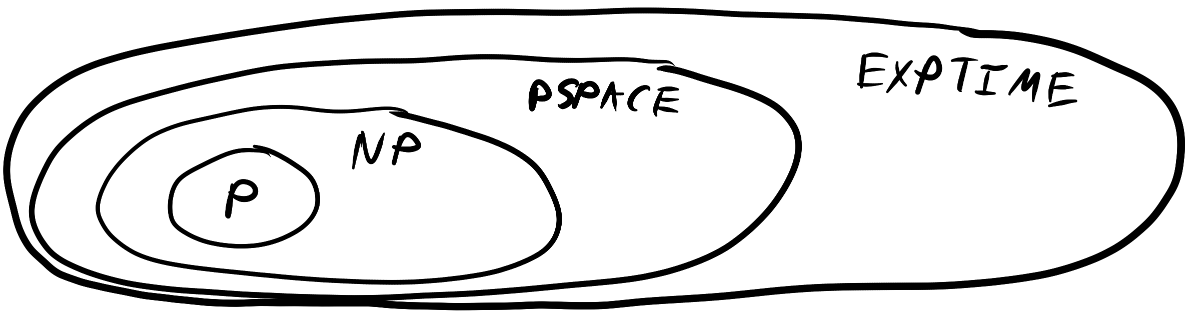
P = problems for which there exist a polynomial-time algorithm

NP = problems whose solutions can be <u>checked</u> in polynomial time

EXPTIME = problems w/ exponential algorithm

PSPACE = problems w/ polynomial <u>space</u> algorithm

Generally we think of P as the class of tractable problems for computation. Clearly $P \subseteq NP$, but whether $P \stackrel{?}{=} NP$ is an open question, and widely believed to be false — that is, NP contains problems which are intractable.



Ex.

- Matrix multiplication is in P ($O(n^3)$ best known)
- Sorting is in P ($O(n \log n)$ best known)
- SAT (determing if a propositional formula is satisfiable) is in NP and is one of the hardest problems in NP
- Factoring an integer into its prime factors is in NP, but not hard for NP. In particular, if it is in P as well, this does NOT imply

$$P = NP$$

(Aside about polynomial time)

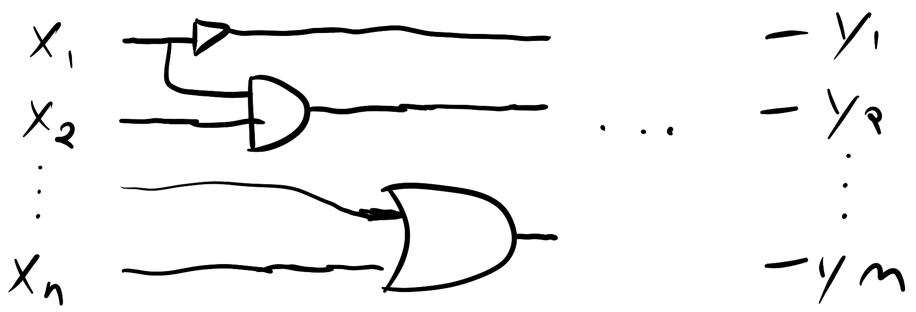
While n^{100} doesn't exactly scream tractable, we use polynomial time to mean tractable or physically realizable computation partially because it works in practice — usually n^k where $1 \leq n \leq 3$, and moreover because we can add and multiply polynomials most differences in computational models come down to polynomial factors, so most models of computation can efficiently simulate one another with respect to P.

Model 1		Model 2
P	$\longleftrightarrow$	P
Not P	$\longleftrightarrow$	Not P

So far we've said nothing formal about the model of computation (only that an algorithm is a python program). To compare classical and quantum computation, it will be easiest to work in the circuit model.

(Classical computation)

Recall from early on that we said in the classical circuit model, a computation is a circuit, e.g.



which computes a function $f : \{0,1\}^n \longrightarrow \{0,1\}^m$

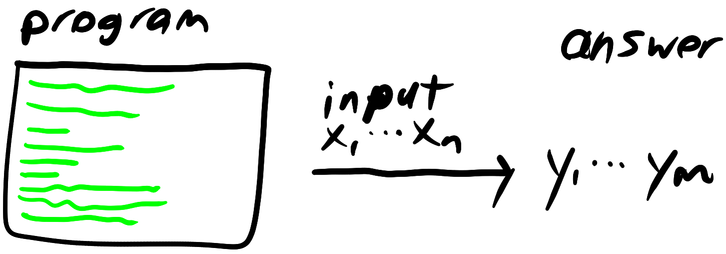
Given a circuit C, we denote the number of gates in C by $|C|$.

(Uniform families of circuits)

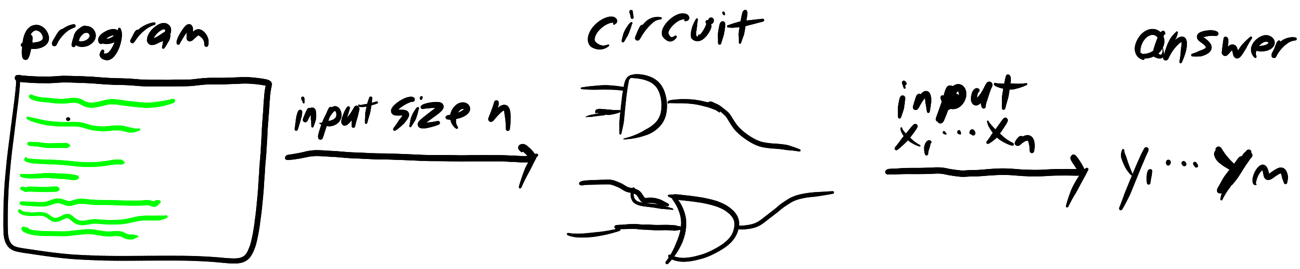
Since a single circuit is fixed – i.e. it has a static and finite number of inputs and outputs, we can't talk about an algorithm or complexity. To do so we need a notion of a circuit family which implements an algorithm with varying input size.

A uniform circuit family is a family of circuits
$$\{ C_n \mid n \in \mathbb{N} \}$$
such that C_n has n input wires and C_n can be generated by an "efficient algorithm" e.g. a python program with runtime polynomial in $|C_n|$ and n

Algorithm

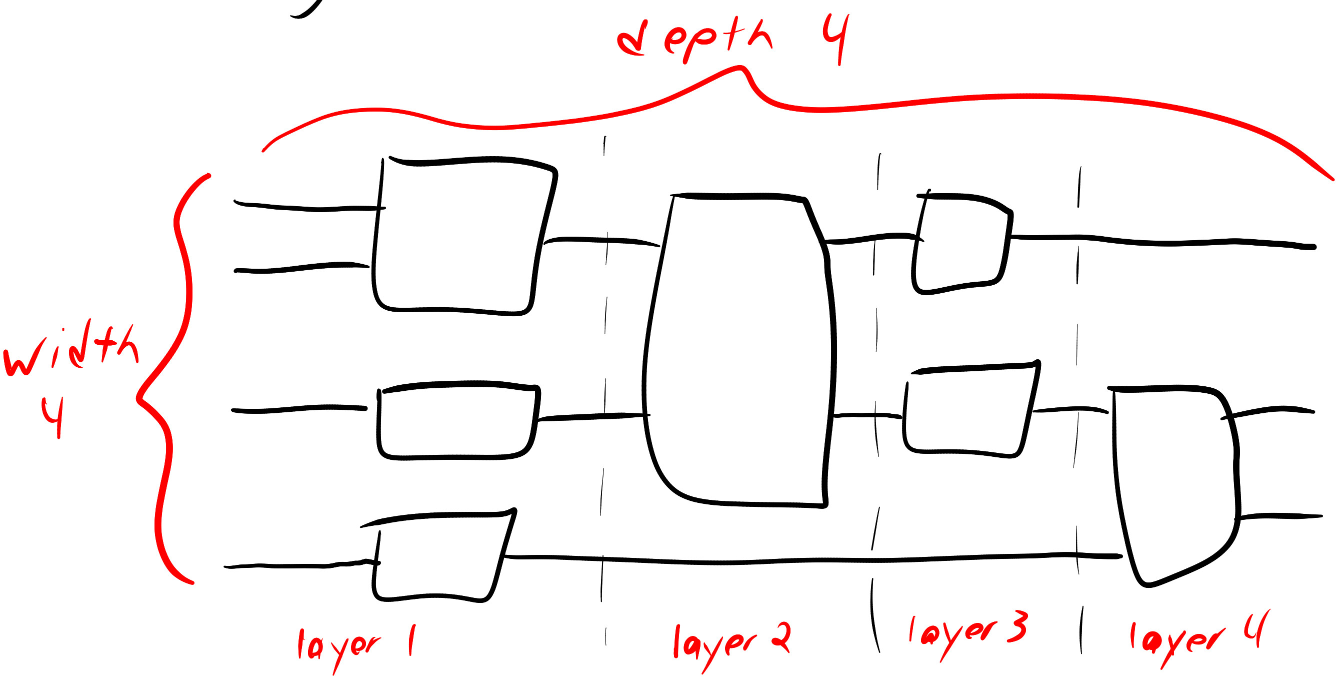


Uniform circuit family



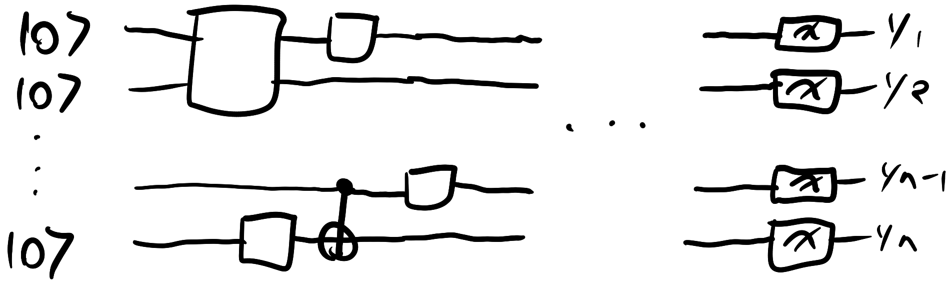
(Circuit complexity)

When we talk about complexity in the circuit model, we typically mean the time or space complexity of a uniform circuit family — that is, the growth rate of the circuit depth or width, respectively, as a function of n. Note that the depth — or the number of "layers" of gates, not the number of gates, gives the time complexity. Usually though, depth and # of gates are polynomially related.



(A quantum circuit model)

Recall that quantum circuits are obtained by replacing bits (e.g. wires) with qubits and classical gates by unitary gates (in particular the same number of inputs & outputs). We further restrict our attention to **deterministic** circuits, which only measure qubits **at the end of a computation.**



We restrict measurement to the **end of computation** because it turns out not to matter (we'll see this with the **principle of deferred measurement**) and the unitary model is more convenient.

(Other models of QC)

Many other models of quantum computation exist, both circuit-like and non-circuit-like. Here are a few:

- Knill's QRAM (quantum random access machine)

$$\text{CPU} \underset{\text{results}}{\overset{\text{instructions}}{\rightleftarrows}} \text{QPU}$$

- Adiabatic QC (e.g. DWAVE)
- Measurement-based QC