MACM 101 **Final exam** **Some Day, 2026**

This is a sample!

Last Name ________________ First Name and Initials ________________

Student No. ___________________________

1. Let compound statements Φ and Ψ be contradictions. Is $\Psi \rightarrow \neg\Phi$ a contradiction? a tautology? neither? Explain your answer.

2. Prove that the sets A and B are disjoint if and only if $A \cup B = A \triangle B$.

3. Prove that the relation $a \equiv b \pmod{m}$ is an equivalence relation on the set of integers.

4. If $f \circ g$ is one-to-one, does it follow that g is one-to-one?

5. Let A and B be countable sets. Show that the set $A \cup B$ is also countable.

6. Prove that for every positive integer n

$$1 \cdot 2^1 + 2 \cdot 2^2 + 3 \cdot 2^3 + \cdots + n \cdot 2^n = (n-1) \cdot 2^{n+1} + 2$$

7. Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

8. Give a proof by contraposition that, for all integers k and l, if kl is odd, then k and l are both odd.

9. Prove that a connected graph is a tree if and only if for any vertices a and b there is a unique path between a and b.

10. Prove that the following relation is an order on the Cartesian product $\{1,2,3\} \times \{1,2,3\}$ and draw its diagram:

$$((x_1, x_2), (y_1, y_2)) \in R \text{ if and only if } x_1 < y_1, \text{ or } x_1 = y_1 \text{ and } x_2 \leq y_2$$

11. State and prove the Chinese Remainder Theorem.

12. Give the definition of the least common multiple lcm(a, b) of two integers a and b.

13. Determine the greatest common divisor of 268 and 400.

14. Suppose a graph G has a cycle of length 7. Can G be bipartite? Explain your answer.

15. Recall that Fibonacci numbers $F(n)$ are given by the following inductive definition:
– $F(1) = F(2) = 1$;
– $F(n+1) = F(n) + F(n-1)$.
Prove that for any natural number $n \geq 2$ the numbers $F(n)$ and $F(n+1)$ are relatively prime.