MACM 101 — Discrete Mathematics I

Exercises on Propositional Logic I.
Complete by: Thursday, May 21st at 11:59pm

Submit your work in pdf format via Crowdmark for a completion mark. If you typed up your work, generate a pdf by, e.g., printing to pdf. If you hand-wrote your solutions, upload a scanned copy in pdf format.

1. There are two tribes living on the island of Knights and Knaves: knights and knaves. You meet three inhabitants of the island of Knights and Knaves, A, B, and C. Let p represents the statement "A is a knight", q represents the statement "B is a knight", and r represents the statement "C is a knight". Use these names along with logic connectives to write each of the following English sentences in symbolic logic notation:

   (a) A is a knave and B or C is a knight.

   (b) A and B are knaves, or A and C are knights.

   (c) At least two people are knights.

   (d) Exactly two people are knights.

2. Find a compound statement involving the propositional variables p, q, and r that is true when p and q are true, and r is false, but false otherwise.

3. There are two tribes living on the island of Knights and Knaves: knights and knaves. Knights always tell truth and knaves always lie. You meet three inhabitants of the island of Knights and Knaves, A, B, and C. A says "B and C are both knaves", B says "Only one of the other two is a knave", and C says "At least one of us is a knave". Who if anyone is a knight?

4. Construct a truth table for the following compound statement: $(p \leftrightarrow q) \rightarrow (q \oplus \neg r)$.

5. Are these system specifications consistent? By consistent we mean **not** a contradiction

   - Whenever the system software is being upgraded, users cannot access the file system.
   - If users can access the file system, then they can save new files.
   - If users cannot save new files, then the system software is not being upgraded.

6. Determine whether the following compound statement is a contradiction

$$((q \vee \neg p) \rightarrow (r \wedge p)) \wedge \neg(r \vee \neg q).$$

7. Use truth tables to check if each of the given pairs of compound statements are equivalent:

   (a) $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$.

   (b) $(\neg q \wedge p) \vee (\neg p \wedge q)$ and $\neg p \vee \neg q$.

   (c) $(p \wedge q) \vee (q \wedge r) \vee (r \wedge p)$ and $(p \vee q) \wedge (q \vee r) \wedge (r \vee p)$.

8. How many of the disjunctions $p \vee \neg q$, $\neg p \vee q$, $q \vee r$, $q \vee \neg r$, and $\neg q \vee \neg r$ can be made simultaneously true by an assignment of truth values to p, q, and r?

9. Use truth tables to determine whether the formula $(p \wedge \neg q) \to (p \wedge q)$ is true whenever $\neg p$ is true.