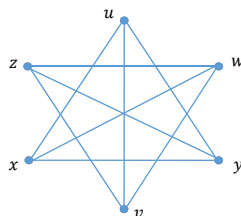


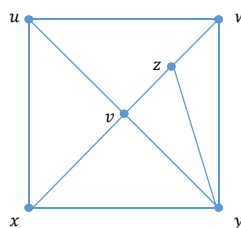
Exercises on Graphs. Complete by: Thursday, August 6th at 11:59pm

Submit your work in pdf format via crowdmark for a completion mark. If you typed up your work, generate a pdf by, e.g., printing to pdf. If you hand-wrote your solutions, upload a scanned copy in pdf format.

1. Let $G = (V, E)$ be a simple graph with $|V| = n$. If $|E| > \left(\frac{n}{2}\right)^2$, prove that G cannot be bipartite.
2. Let $G = (V, E)$ be a simple connected 4-regular planar graph. If $|E| = 16$, how many regions are there in a planar representation of G ?
3. Is the following graph planar? If yes, find the number of regions in its planar representation? Explain your answer.



4. Find an adjacency list, adjacency matrix, and an incidence matrix of the following graph

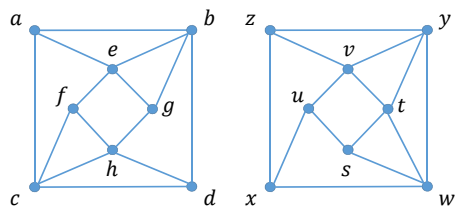


5. Draw (multi-) (di-)graphs given by the following adjacency and inci-

dence matrices

$$\begin{pmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{pmatrix} \quad \begin{pmatrix} 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \end{pmatrix}.$$

6. Are the following graphs isomorphic? Explain.



7. Let $G = (V, E)$ be an undirected graph. Define a binary relation R on V by $(u, v) \in R$ if and only if $u = v$ or there is a path from u to v . Prove that R is an equivalence relation. What are its equivalence classes?
8. Show that every connected graph with n vertices has at least $n - 1$ edges.
9. Let $G = (V, E)$ be a simple connected undirected graph, where $V = \{v_1, v_2, v_3, \dots, v_n\}$, $n \geq 2$, $\deg(v_1) = 1$, and $\deg(v_i) \geq 2$ for $2 \leq i \leq n$. Prove that G has a cycle.
10. Let $T = (V, E)$ be a tree where $|V| = n$. Suppose that for each $v \in V$ either $\deg(v) = 1$ or $\deg(v) \geq m$, where m is a fixed positive integer, $m \geq 2$, and there is a vertex of degree at least m . What is the smallest value possible for n .