MACM 101 — Discrete Mathematics I

Exercises on Propositional Logic II.
Complete by: Thursday, May 28th at 11:59pm

Submit your work in pdf format via crowdmark for a completion mark. If you typed up your work, generate a pdf by, e.g., printing to pdf. If you hand-wrote your solutions, upload a scanned copy in pdf format.

1. Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent. Do not use truth tables.

2. Simplify the compound statement

$$((((p \wedge q) \wedge r) \vee ((p \wedge q) \wedge \neg r)) \vee \neg q) \to s.$$

3. Are the following compound statements equivalent? Do not use truth tables.

$$(s \to (\neg p \vee r)) \wedge ((p \to (r \vee q)) \wedge s), \quad \text{and} \quad s \wedge q.$$

4. Let NAND be the logic connective defined by $p \uparrow q \Leftrightarrow \neg(p \wedge q)$. Show that $p \uparrow (q \uparrow r)$ is not logically equivalent to $(p \uparrow q) \uparrow r$.

5. Prove that the Rule of Proof by Cases is a valid argument.

6. Each of two rooms (room I and room II) contains either a lady or a tiger. If a room contains a lady, the sign on its door is true. If it contains a tiger, the sign is false. The signs are

I	II
AT LEAST ONE ROOM CONTAINS A LADY	THE OTHER ROOM CONTAINS A LADY

   Which rooms contain ladies? Use logic equivalences or rules of inference.

7. What relevant conclusion or conclusions can be drawn from this set of premises? Explain the rules of inference used to obtain each conclusion from the premises.
   "If I eat spicy foods, then I have strange dreams."
   "I have strange dreams if there is thunder while I sleep."
   "I did not have strange dreams."

8. Write the following argument in symbolic form. Then establish the validity of the argument or give a counterexample to show that it is invalid.

If Dominic goes to the racetrack, then Helen will be mad. If Ralph plays cards all night, then Carmela will be mad. If either Helen or Carmela gets mad, then Veronica (their attorney) will be notified. Veronica has not heard from either of these two clients. Consequently, Dominic didn't make it to the racetrack and Ralph didn't play cards all night.

9. Write the following argument in symbolic form. Then establish the validity of the argument or give a counterexample to show that it is invalid.

   If Newton is not considered a great mathematician and Leibniz's work is not ignored, then calculus would not be the centerpiece of the modern math curriculum. Newton is considered the greatest mathematician only if Leibniz's work is ignored. Therefore, calculus is the centerpieceof the modern math curriculum and Leibniz's work is not ignored.

10. Using rules of inference and logic equivalences give the reasons for the steps verifying the following argument.
    Premises: $(t \vee q) \rightarrow (p \wedge \neg r)$, $(u \vee \neg s) \rightarrow t$, $(p \wedge \neg r) \rightarrow q$, $\neg s \vee t \vee q$, $\neg t \wedge u$, $p \vee \neg q$.
    Conclusion: p.

Steps	Reasons
1) $\neg t \wedge u$	
2) $\neg t$	
3) $(u \vee \neg s) \rightarrow t$	
4) $\neg t \rightarrow \neg(u \vee \neg s)$	
5) $\neg t \rightarrow (\neg u \wedge s)$	
6) $\neg u \wedge s$	
7) s	
8) $(t \vee q) \rightarrow (p \wedge \neg r)$	
9) $\neg s \vee t \vee q$	
10) $s \rightarrow (t \vee q)$	
11) $s \rightarrow (p \wedge \neg r)$	
12) $p \wedge \neg r$	
13) $(p \wedge \neg r) \rightarrow q$	
14) q	
15) $p \vee \neg q$	
16) p	