MACM 101 — Discrete Mathematics I

Exercises on Predicates and Quantifiers.
Complete by: Thursday, June 4th at 11:59pm

Submit your work in pdf format via crowdmark for a completion mark. If you typed up your work, generate a pdf by, e.g., printing to pdf. If you hand-wrote your solutions, upload a scanned copy in pdf format.

1. Write each of the following statements using quantifiers and predicates. In each case, you must specify the domain and define the predicates you use.

   (a) There are computer science majors who do not minor in mathematics.

   (b) Every two real numbers have an integer in between.

   (c) No square number immediately follow a prime number.

2. Let $F(x, y)$ be the statement "x can fool y", where the domain consists of all people in the world. Use quantifiers to express each of these statements.

   (a) Everybody can fool somebody.

   (b) There is no one who can fool everybody.

   (c) Everyone can be fooled by somebody.

   (d) No one can fool both Fred and Jerry.

   (e) Nancy can fool exactly two people.

3. Let the domain be the set $D = \{1, 3, 5, 7, 8, 10, 11, 12\}$. Write each of the following statements using predicates and quantifiers, and decide whether it is true for all the elements from D. If it is not, give a counterexample.

   (a) If x is even and $x > 7$, then $x < 20$.

   (b) If x is odd, then $x < 10$.

   (c) If x is odd or $x < 5$, then $x - 1$ is even.

4. Determine the truth value of each of the following statements if the domain of each variable consists of all real numbers.

   (a) $\exists x\ \forall y\ (y \neq 0 \rightarrow (xy = 1))$;

   (b) $\forall x \forall y \exists z\ (z = \frac{x+y}{2})$.

5. Use predicates and quantifiers to express the following statement

   "Some students in this class grew up in the same town as at least two other students in this class."

6. Find a counterexample, if possible, to the following universally quantified statement, where the domain for each variable consists of all integers

$$\forall x \exists y \ (2x + 5y \leq x + y).$$

7. Express the negation of the following statements using implication (if... then...)

   (a) There exists a positive integer n such that n is even and $\frac{1}{n} > 1$.

   (b) There exist positive integers a and b such that $a - b$ is odd and $a^2 = 2b^2$.

8. Find a domain for variables x, y, and z in which the statement

$$\exists x \exists y \forall z ((x = z) \vee (y = z))$$

   is true and another domain in which it is false.