MACM 101 — Discrete Mathematics I

Exercises on Predicates and Quantifiers II.
Complete by: Thursday, June 11th at 11:59pm

Submit your work in pdf format via crowdmark for a completion mark. If you typed up your work, generate a pdf by, e.g., printing to pdf. If you hand-wrote your solutions, upload a scanned copy in pdf format.

1. Rewrite the following statement so that negations appear only within predicates (that is, no negation is outside a quantifier or an expression involving logical connectives)

$$\neg\forall x\ ((\forall y \exists z\ P(x,y,z)) \oplus (\forall z \forall y\ (R(x,y,z) \vee S(z,y)))).$$

2. Prove that quantified statements

$$\forall x(P(x) \oplus Q(x)) \quad \text{and} \quad (\forall x P(x)) \oplus (\forall x Q(x))$$

   are not logically equivalent.

3. Prove that quantified statements

$$\forall x \forall y \exists z P(x,y,z) \quad \text{and} \quad \forall x \exists z \forall y P(x,y,z)$$

   are not logically equivalent.

4. Prove that the statements

$$\forall x \forall y P(x,y) \quad \text{and} \quad \forall x \forall y P(y,x)$$

   are logically equivalent, assuming both indeterminates have the same domain.

5. Assume that $\exists x \forall y P(x,y)$ is true and that the domain for x, y is nonempty. Which of the following must be true?

$$\forall x \exists y P(x,y) \quad \text{and} \quad \exists x \exists y P(x,y).$$

   Prove your answer either using a regular proof in the natural language or by giving a formal inference. In the latter case you may need to use the existential rules of inference.

6. Determine whether the following argument is valid or invalid and explain why by giving a formal inference if the argument is valid, or by explaining why a certain step in the argument is incorrect or providing a counterexample if the argument is invalid.

   'Everyone who eats granola every day is healthy.'
   'Linda is not healthy.'
   'Therefore, Linda does not eat granola every day.'

7. Determine whether the following argument is valid or invalid and explain why by giving a formal inference if the argument is valid, or by explaining why a certain step in the argument is incorrect or providing a counterexample if the argument is invalid.

   'All discrete mathematics students can tell a valid argument from invalid,'
   'All thoughtful people can tell a valid argument from invalid,'
   'Therefore, all discrete mathematics students are thoughtful'

8. Given premises:

   'All hummingbirds are richly colored.'
   'No large birds live on honey.'
   'Birds that do not live on honey are dull in color.'

   infer the conclusion

   'Hummingbirds are small.'

9. By providing a formal inference justify the rule of universal transitivity:
   if $\forall x(P(x) \to Q(x))$ and $\forall x(Q(x) \to R(x))$ are true, then $\forall x(P(x) \to R(x))$ is also true.

10. What is wrong with this proof?

    Theorem. 7 is divisible by 3

    Proof. Every integer number is divisible by 3 or it is not. Let c be an arbitrary integer number. Therefore, it is divisible by 3 or it is not. Suppose it is divisible by 3. By the rule of universal generalization, if an arbitrary number is divisible by 3, every number is is divisible by 3. Therefore, 7 is divisible by 3.

11. Prove that the sum of an irrational number and a rational number is irrational.