MACM 101 — Discrete Mathematics I

Exercises on Sets. Complete by: Thursday, June 25th at 11:59pm

Submit your work in pdf format via crowdmark for a completion mark. If you typed up your work, generate a pdf by, e.g., printing to pdf. If you hand-wrote your solutions, upload a scanned copy in pdf format.

1. Let $A = \{x \in \mathbb{Z} \mid x = 5a + 2 \text{ for some integer } a\}$,
   $B = \{y \in \mathbb{Z} \mid y = 10b - 3 \text{ for some integer } b\}$,
   $C = \{z \in \mathbb{Z} \mid z = 10c + 7 \text{ for some integer } c\}$,
   Prove or disprove each of the following statements:

   a. $A \subseteq B$ b. $B \subseteq A$ c. $B = C$

2. Let the universe be the set $\mathbb{R}$ of all real numbers and let
   $A = \{x \in \mathbb{R} \mid 0 < x \leq 2\}$, $B = \{x \in \mathbb{R} \mid 1 \leq x < 4\}$,
   $C = \{x \in \mathbb{R} \mid 3 \leq x < 9\}$.
   Find each of the following:

   a. $A \cup B$ b. $A \cap B$ c. $\overline{A}$ d. $A \cup C$
   e. $A \cap C$ f. $\overline{B}$ g. $\overline{A} \cap \overline{B}$
   h. $\overline{A} \cup \overline{B}$ i. $\overline{A \cap B}$ j. $\overline{A \cup B}$

3. Find $\mathcal{P}(\mathcal{P}(\mathcal{P}(\emptyset)))$, where $\mathcal{P}(A)$ denotes the power set of A.

4. Using laws of set theory show that

$$\overline{(A \cup B) \cap C} = (\overline{A} \cup \overline{C}) \cap (\overline{B} \cup \overline{C}).$$

5. Let A, B, and C be sets. Show that

$$(A - C) \cap (C - B) = \emptyset.$$

   Draw Venn diagrams for the left hand side expression.

6. What can you say about the sets A and B if we know that $A - B = A$? Prove your answer.

7. Prove that for all sets A, B, C, if $B \cap C \subseteq A$, then $(C - A) \cap (B - A) = \emptyset$.

8. Show that for any sets A and B

$$A \triangle B = \overline{A} \triangle \overline{B}.$$

   Draw Venn diagrams for both expressions.

9. Prove that

$$A \times B \times (A \cap C) = (A \times B \times A) \cap (A \times B \times C).$$

10. What is wrong with the following proof? You do not need to write down a formal inference or show a wrongly used rule of inference. Just point a step in the argument that is not valid and explain why.

    Theorem. For all sets A, B, $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$.

    Proof. Suppose A and B are sets, and $x \in \overline{A} \cup \overline{B}$. Then $x \in \overline{A}$ or $x \in \overline{B}$ by definition of union. It follows that $x \notin A$ or $x \notin B$ by definition of complement, and so $x \notin A \cup B$ by definition of union. Thus $x \in \overline{A \cup B}$, and hence $\overline{A} \cup \overline{B} \subseteq \overline{A \cup B}$.