MACM 101 — Discrete Mathematics I

Exercises on Relations. Complete by: Thursday, July 2nd at 11:59pm

Submit your work in pdf format via crowdmark for a completion mark. If you typed up your work, generate a pdf by, e.g., printing to pdf. If you hand-wrote your solutions, upload a scanned copy in pdf format.

1. Make a list of pairs, construct the matrix, and draw the graph of the relation R from the set $A = \{0, 1, 2, 3, 4\}$ to the set $B = \{0, 1, 2, 3\}$ such that $(a, b) \in R$ if and only if $a + b = 4$.

2. Let R be a binary relation on the set of all positive integers given by $(x, y) \in R$ if and only if $xy \geq 1$. Determine if this relation reflexive, symmetric, transitive, or antisymmetric. Prove your answer.

3. Let X be the set of all 4-bit strings (e.g. 0011,0101,1000, etc.). Define a relation R on X as $(s, t) \in R$ if and only if some substring of s of length 2 is equal to some substring of t of length 2. For example, $(0111, 0101) \in R$ because both 0111 and 0101 contain 01; however, $(1110, 0001) \notin R$ because 1110 and 0001 do not share a common substring of length 2. Is this relation reflexive, symmetric, transitive, or antisymmetric? Prove your answer.

4. Construct a relation on the set $\{1, 2, 3, 4\}$ that is reflexive, antisymmetric, and not transitive.

5. Recall that binary relations are merely sets of pairs of elements of some set. Therefore if R and Q are binary relations on a same set A, their union $R \cup Q$ as sets of pairs is also a binary relation on A. Is it true that if R and Q are antisymmetric then $R \cup Q$ is also antisymmetric? Prove that it is or give a counterexample.

6. Let R be the relation on $\{1, 2, 3\} \times \{1, 2, 3\}$, that is, elements of this relation are pairs of pairs of integers, such that $((a, b), (c, d)) \in R$ if and only if $a + d = b + c$. Show that R is an equivalence relation. Find its equivalence classes.

7. Let f be a function from A to B. Define a relation R on A by $(x, y) \in R$ if and only if $f(x) = f(y)$. Prove that R is an equivalence relation.

8. Relation R is given by matrix

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 \end{pmatrix}.$$

   Is R an order? If yes, what its minimal, maximal, least, and greatest elements are?

9. Let $A = \{1, 2, 3, 4\}$, and let R be a binary relation on $A \times A$ given by: $((a, b), (c, d)) \in R$ if and only if $a \leq c$ and $b \leq d$. Show that R is an order and draw its diagram.

10. Let R be a relation that is symmetric and antisymmetric. Show that R is transitive.