MACM 101 — Discrete Mathematics I

Exercises on Integers. Complete by: Thursday, July 30th at 11:59pm

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1. Find the binary (base 2), octal (base 8), and hexadecimal (base 16) expansion of 1234567.

2. Add the hexadecimal numbers $49F7 + C66$.

3. Find the greatest common divisor, the least common multiple of $a = 2475$ and $b = 32670$. Find numbers u, v such that $gcd(a, b) = ua + vb$.

4. Prove that for any positive integers a, b it holds that $gcd(a, b) = gcd(a, a+b)$.

5. Is it true that for any odd n the number $2^n + 1$ is divisible by 3? Prove your answer.

6. Prove that there are no 3 consecutive odd prime numbers, that is, for any odd p, at least one of $p, p + 2, p + 4$ is composite.

7. Show that if a, b, and n are integers such that $n \geq 2$ and $a \equiv b \pmod{n}$, then $gcd(a, n) = gcd(b, n)$.

8. Let m, n be positive integers, and let $f : \mathbb{Z}_m \to \mathbb{Z}_m$ be the function defined by $f(x) = nx \pmod{m}$. Prove that f is one-to-one if and only if $gcd(m, n) = 1$.

9. Find the inverse of 11 modulo 47.

10. Solve the congruence $3x \equiv 7 \pmod{19}$.