

Laws of Logic

Aministrivia

Tutorials/Labs

- Not required, but strongly recommended

Homework

- Crowdmark isn't set up yet (but it will be 😊)

Quizzes

- First is on Friday in the **first 10 minutes of lecture**
- Similar format to homework/practice exercises

Lecture recordings

- IT managed
- They had an error on Friday
- In general, **don't count on recordings being there**

Previous Lecture

- Truth tables
- Tautologies and contradictions
- Logic equivalences

Laws of Logic

● Double negation

$$\neg\neg p \Leftrightarrow p$$

p	$\neg p$	$\neg\neg p$
0	1	0
1	0	1



Laws of Logic (cntd)

● DeMorgan's laws

$$\neg(p \wedge q) \Leftrightarrow \neg p \vee \neg q$$

$$\neg(p \vee q) \Leftrightarrow \neg p \wedge \neg q$$

p	q	$\neg p$	$\neg q$	$p \wedge q$	$\neg(p \wedge q)$	$\neg p \vee \neg q$
0	0	1	1	0	1	1
0	1	1	0	0	1	1
1	0	0	1	0	1	1
1	1	0	0	1	0	0


Example

- Construct the negation of

‘Miguel has a cell phone and he has a laptop’


‘Heather will go to the concert or Steve will go to the concert’

'Algebraic' Laws of Logic




$$p \wedge q \Leftrightarrow q \wedge p$$

$$\underline{p \vee q} \Leftrightarrow q \vee p$$
Commutative laws



$$\underline{p \wedge (q \wedge r)} \Leftrightarrow (p \wedge q) \wedge r$$

$$\underline{p \vee (q \vee r)} \Leftrightarrow (p \vee q) \vee r$$
Associative laws



$$\underline{p \wedge (q \vee r)} \Leftrightarrow (\underline{p \wedge q}) \vee (\underline{p \wedge r})$$

$$\underline{p \vee (q \wedge r)} \Leftrightarrow (p \vee q) \wedge (p \vee r)$$
Distributive laws



$$\underline{p \wedge p} \Leftrightarrow p$$

$$\underline{p \vee p} \Leftrightarrow p$$
Idempotent laws

'Logic' Laws of Logic

$$\bullet \quad \begin{aligned} p \wedge T &\Leftrightarrow p \\ p \vee F &\Leftrightarrow p \end{aligned}$$

Identity laws

$$\bullet \quad \begin{aligned} p \wedge \neg p &\Leftrightarrow F \\ p \vee \neg p &\Leftrightarrow T \end{aligned}$$

Inverse laws

the law of contradiction
the law of excluded middle

$$\bullet \quad \begin{aligned} p \wedge F &\Leftrightarrow F \\ p \vee T &\Leftrightarrow T \end{aligned}$$

Domination/annihilation laws

$$\bullet \quad \begin{aligned} p \wedge (p \vee q) &\Leftrightarrow p \\ p \vee (p \wedge q) &\Leftrightarrow p \end{aligned}$$

Absorption laws

Example



- Simplify the proposition

$$\neg(q \vee r) \vee \neg(\neg q \vee p) \vee (r \vee p)$$

(DeMorgan's) $\Leftrightarrow (\neg q \wedge \neg r) \vee (\neg \neg q \wedge \neg p) \vee (r \vee p)$

$$\Leftrightarrow (\neg q \wedge \neg r) \vee (q \wedge \neg p) \vee (r \vee p)$$

$$\Leftrightarrow (\neg q \wedge \neg r) \vee r \vee (q \wedge \neg p) \vee p$$

$$\Leftrightarrow ((\neg q \vee r) \wedge (\neg r \vee r)) \vee ((q \vee p) \wedge (\neg p \vee p))$$

$$\Leftrightarrow (\neg q \vee r) \vee (q \wedge p)$$

Derived laws $(a+b)(c+d) = ac + ad + bc + bd$

- Some laws can be obtained by combining other laws, such as the **double distributive laws**

$$\begin{aligned} \underline{(p \vee q) \wedge (r \vee s)} &\Leftrightarrow (p \wedge r) \vee (p \wedge s) \vee (q \wedge r) \vee (q \wedge s) \\ (p \wedge q) \vee (r \wedge s) &\Leftrightarrow (p \vee r) \wedge (p \vee s) \wedge (q \vee r) \wedge (q \vee s) \end{aligned}$$

$$\begin{aligned} (p \vee q) \wedge (r \vee s) &\Leftrightarrow (p \wedge (r \vee s)) \vee (q \wedge (r \vee s)) \\ &\Leftrightarrow (p \wedge r) \vee (p \wedge s) \vee (q \wedge r) \vee (q \wedge s) \end{aligned}$$

Order of operations

● \neg binds most tightly

● Then \wedge

● Then \vee, \oplus

● Next \rightarrow

● Finally, \leftrightarrow

Example: parenthesize correctly

$$((p \wedge q) \rightarrow r) \leftrightarrow ((\neg p) \vee (\neg q) \vee r)$$

Typically write multiple \vee, \wedge **without parentheses**

$$p \wedge (q \wedge r) \Leftrightarrow p \wedge q \wedge r \Leftrightarrow (p \wedge q) \wedge r$$

$$p \vee (q \vee r) \Leftrightarrow p \vee q \vee r \Leftrightarrow (p \vee q) \vee r$$

By associativity

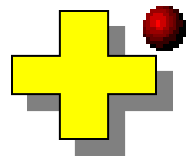
Expressing Connectives

● Some connectives can be expressed through others

■ $p \oplus q \Leftrightarrow \neg(p \leftrightarrow q)$

■ $p \leftrightarrow q \Leftrightarrow (p \rightarrow q) \wedge (q \rightarrow p)$

■ $p \rightarrow q \Leftrightarrow \neg p \vee q$



Theorem Every compound proposition is logically equivalent to a proposition that uses only conjunction, disjunction, and negation

Example

“If you are a computer science major or a freshman and you are not a computer science major or you are granted access to the Internet, then you are a freshman or have access to the Internet”

p - ‘you can access the Internet’

q - ‘you are a computer science major’

r - ‘you are a freshman’

$$(q \vee r) \wedge (\neg q \vee p) \rightarrow (r \vee p)$$

$$\Leftrightarrow \neg((q \vee r) \wedge (\neg q \vee p)) \vee r \vee p$$

$$\Leftrightarrow \neg(q \vee r) \vee \neg(\neg q \vee p) \vee r \vee p$$

Example

- Simplify the proposition

$$(p \vee q) \leftrightarrow (p \wedge q)$$

$$\Leftrightarrow (p \vee q) \rightarrow (p \wedge q) \wedge (p \wedge q) \rightarrow (p \vee q)$$

$$\Leftrightarrow (\neg(p \vee q) \vee (p \wedge q)) \wedge (\neg(p \wedge q) \vee (p \vee q))$$

$$\Leftrightarrow ((\neg p \wedge \neg q) \vee (p \wedge q)) \wedge ((\neg p \vee \neg q) \vee (p \vee q))$$

$$\Leftrightarrow ((\neg p \vee p) \wedge (\neg p \vee q) \wedge (\neg q \vee p) \wedge (\neg q \vee q)) \wedge \text{" "}$$

$$\Leftrightarrow ((\neg p \vee q) \wedge (\neg q \vee p)) \wedge \text{" "}$$

First Law of Substitution

- Suppose that Φ is a tautology, and p is a primitive proposition.
- Let Φ' be the formula Φ with each occurrence of p replaced by a formula Ψ
 - sometimes denoted $\Phi[\Psi / p]$
- Then Φ' is a tautology

Example:

Let $\Phi = (p \rightarrow q) \vee (q \rightarrow p)$ and $\Psi = p \vee (s \oplus r)$

Then

$\Phi' = \Phi[\Psi / p] = ((p \vee (s \oplus r)) \rightarrow q) \vee (q \rightarrow (p \vee (s \oplus r)))$
is a tautology

Second Law of Substitution

- Let Φ be a compound proposition and Ψ be a subformula of Φ
- Suppose $\Psi \Leftrightarrow \Psi'$
- Then replacing Ψ with Ψ' in Φ yields a **logically equivalent formula**

Example:

Let $\Phi = (p \rightarrow q) \vee (q \rightarrow p)$ and $\Psi = p$

Set $\Psi' = p \vee (p \wedge q)$

Recall that $p \Leftrightarrow p \vee (p \wedge q)$ by Absorption Law...

Then

$$\underline{(p \rightarrow q)} \vee (q \rightarrow p) \Leftrightarrow \underline{\underline{((p \vee (p \wedge q)) \rightarrow q)}} \vee (q \rightarrow p).$$

Practice

Exercises from the Book:

7th edition: No. 27, 5, 11(c,d) without truth tables, 55 (page 34)

8th edition: No. 31, 5, 11(c,d) without truth tables, 59 (page 38)

- Express conjunction and disjunction through implication and negation
(*)

Practice

- Show that $(p \vee q) \rightarrow r$ and $(p \rightarrow r) \vee (q \rightarrow r)$ are not logically equivalent

- Is the following argument valid?

If the band could not play rock music or refreshments were not delivered on time, then the New Year's party would have been canceled and Alicia would have been angry. If the party were canceled, then refunds would have to be made. No refunds were made.

Therefore the band could play rock music