

Rules of Inference

Logical inference

- *A type of logical argument*

- *Given a collection of **premises**, deduce a **conclusion***

"There is blood on Zora's knife. Whoever's knife has blood on it is the killer. Therefore, Zora is the killer"

- *An inference is **valid** if whenever each of the premises is true the conclusion is also true*

"If pigs fly, then I can fly. Pigs fly. Therefore I can fly."

- ***Rules of inference** give small, **valid** patterns of inference which we can use to build larger **arguments***

Example

Consider the following arguments:

*“If you have a current password, then you can log onto the network.
You have a current password. Therefore, you can log onto the
network.”*

and

*“If you have a current password, then you can log onto the network.
You can log onto the network. Therefore, you have a current
password.”*

Inference in symbolic form

- Write these arguments in symbolic form

p - you have a current password

q - you can log onto the network

$$\frac{p \rightarrow q}{p}$$

$\therefore q$

$$\frac{p \rightarrow q}{q}$$

$\therefore p$

‘Therefore’



Inference and Tautologies

- If both $p \rightarrow q$ and p are true, since the implication $p \rightarrow q$ is true and p is true, q must also be true.
- Equivalently, $\Phi = ((p \rightarrow q) \wedge p) \rightarrow q$ is a tautology:

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge p$	Φ
0	0	1		
0	1	1		
1	0	0		
1	1	1		

The first example is a **valid** argument!

Inference and Tautologies

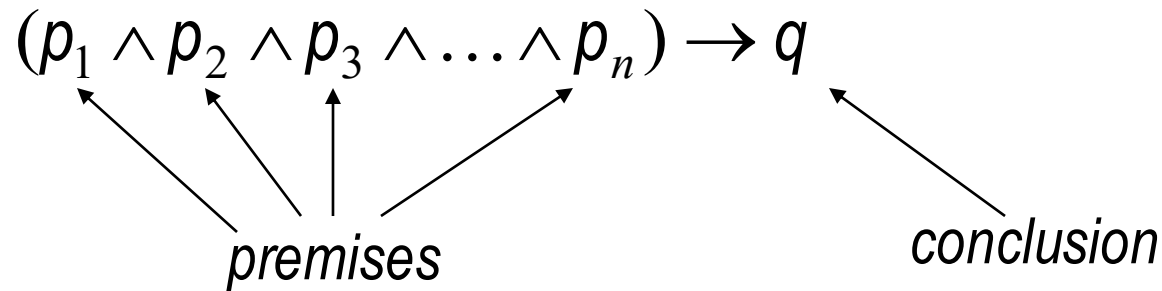
● Let us try $\Psi = ((p \rightarrow q) \wedge q) \rightarrow p$

p	q	$p \rightarrow q$	$(p \rightarrow q) \wedge q$	Ψ
0	0	1		
0	1	1		
1	0	0		
1	1	1		

In the case $p = 0$, $q = 1$ both conditions $(p \rightarrow q)$ and q are true, but p is false. This is not a valid argument! It is a **logical fallacy**

General Definition of Inference

- *The general form of an inference in symbolic form is*



- *The inference is **valid** if whenever each of the premises is true the conclusion is also true*
- *Equivalently, the inference is valid if and only if the following compound proposition is a tautology*

$$(p_1 \wedge p_2 \wedge p_3 \wedge \dots \wedge p_n) \rightarrow q$$

Rules of Inference

- *Checking that a complicated inference is valid is generally hard, even for computer.*
- *Instead, build complicated valid arguments by breaking down into known, valid **rules of inference**.*

● *E.g., Modus ponens*

$$\begin{array}{l} p \rightarrow q \\ p \\ \hline \therefore q \end{array}$$

*“If you have a current password, then you can log onto the network.
You have a current password.
Therefore, you can log onto the network.”*

Rule of Syllogism

$$\begin{array}{l} \bullet \quad p \rightarrow q \\ \quad q \rightarrow r \\ \hline \therefore p \rightarrow r \end{array}$$

• The corresponding tautology $((p \rightarrow q) \wedge (q \rightarrow r)) \rightarrow (p \rightarrow r)$

*“If you send me an e-mail, then I’ll finish writing the program.
If I finish writing the program, then I’ll go to sleep early.”*

p - ‘you will send me an e-mail’

q - ‘I will finish writing the program’

r - ‘I will go to sleep early’

“Therefore, if you send me an e-mail, then I’ll go to sleep early”

Modus Tollens

$$\begin{array}{l} \bullet \quad p \rightarrow q \\ \quad \neg q \\ \hline \therefore \neg p \end{array}$$

• The corresponding tautology $((p \rightarrow q) \wedge \neg q) \rightarrow \neg p$

“If today is Friday, then tomorrow I’ll go skiing”.

“I will not go skiing tomorrow”.

p - ‘today is Friday’

q - ‘I will go skiing tomorrow’

“Therefore, today is not Friday”

Rule of Disjunctive Syllogism

$$\begin{array}{l} \bullet \quad p \vee q \\ \quad \neg p \\ \hline \therefore q \end{array}$$

• The corresponding tautology $((p \vee q) \wedge \neg p) \rightarrow q$

“I’ll go skiing this weekend.

I will not go skiing on Saturday.”

p - ‘I will go skiing on Saturday’

q - ‘I will go skiing on Sunday’

“Therefore, I will go skiing on Sunday”

Rule for Proof by Cases

$$\begin{array}{l} \bullet \quad p \rightarrow r \\ \quad q \rightarrow r \\ \hline \therefore (p \vee q) \rightarrow r \end{array}$$

• The corresponding tautology $((p \rightarrow r) \wedge (q \rightarrow r)) \rightarrow ((p \vee q) \rightarrow r)$

“If today is Saturday, then I’ll go skiing.

If today is Sunday, then I’ll go skiing.

p - ‘today is Saturday’

q - ‘today is Sunday’

r - ‘I’ll go skiing’

“Therefore, if today is Saturday or Sunday, then I will go skiing”

Rules of Contradiction, Simplification, and Amplification

● *Rule of Contradiction* *Reductio ad Absurdum*

$$\frac{p \rightarrow F}{\therefore \neg p} \quad \text{The corresponding tautology } (p \rightarrow F) \rightarrow \neg p$$

● *Rule of Simplification*

$$\frac{p \wedge q}{\therefore p} \quad \text{The corresponding tautology } (p \wedge q) \rightarrow p$$

● *Rule of Addition*

$$\frac{p}{\therefore p \vee q} \quad \text{The corresponding tautology } p \rightarrow (p \vee q)$$

Fallacies

● *Affirming the conclusion*

$$\frac{p \rightarrow q}{q} \quad \text{The corresponding proposition}$$

$$\therefore p \quad ((p \rightarrow q) \wedge q) \rightarrow p$$

● *Denying the hypothesis*

$$\frac{p \rightarrow q}{\neg p} \quad \text{The corresponding proposition}$$

$$\therefore \neg q \quad ((p \rightarrow q) \wedge \neg p) \rightarrow \neg q$$

Practice

Exercises from the Book:

7th edition: No. 3, 5, 6, 11, 12 (page 78)

8th edition: No. 3, 5, 6, 11, 12 (page 82)