

# Midterm 1 review

# Propositional logic

● *Logical connectives*

$\wedge, \vee, \neg, \oplus, \rightarrow, \leftrightarrow$

● *Truth tables*

● *Definitions of tautology, contradiction, contrapositive, and logical equivalence*

● *Logical equivalences*

- *Equivalent expressions for connectives*
- *Logical laws like De Morgan's laws and distributivity*

● *Rules of inference*

● *Logical arguments*

- *How to write them*
- *Annotate with reasons*

# Predicates and quantifiers

- *How to write statements using predicates & quantifiers*
- *Meaning of quantified formulas*
- *Definition of logical equivalence*
  - *For all interpretations of the predicates*
  - *How to prove or disprove logical equivalence*
- *Logical laws for quantified statements*
  - *De Morgan's laws for quantifiers*
  - *Applying (propositional) equivalences inside quantifiers*
- *Rules of inference*
  - *Instantiation and generalization rules*
  - *How to use in logical arguments*

$\exists x. P(x) \rightarrow Q(x)$   
 $P(\text{Bob})$

# Proof strategies

- *(Will not factor heavily into midterm)*
- *Direct proofs*
  - *Follow the structure of the formula & associated rule of inference*
- *Logical equivalence*
  - *Prove a logically equivalent statement*
  - *Notably, contrapositive proofs*
- *Proof by cases*
- *Proof by contradiction*

# Sets



## ● Basic sets

- Integers  $\mathbb{Z} = \{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$
- Real numbers  $\mathbb{R} = \{\dots, 0.9, 0.99, 0.999, \dots\}$

## • Set builder notation

## ● Definition of cardinality

## ● Operations on sets

- Intersection
- Union
- Complement
- Difference
- Symmetric difference

## • Power sets

## Tips on formal inferences

- *When doing formal inferences/arguments, it's a matching game*
- *Try to*
  - *Combine premises to derive new facts*
  - *Use all of the premises*
  - *If you do that, you'll probably derive the conclusion*
- *How do we figure out when we can combine premises?*
  - *Suppose we have a premise the  $P(a)$  is true*
  - *Look for an implication or disjunction involve  $P$ , potentially quantified*
  - *$P(a) \rightarrow Q(a)$ , apply modus ponens*
  - *$Q(a) \rightarrow \neg P(a)$ , apply modus tollens*
  - *$\neg P(a) \vee Q(a)$ , apply disjunction syllogism / resolution*
- *What about other types of premises?*
  - *$P(a) \wedge Q(a)$ , derive  $P(a)$  and/or  $Q(a)$*
  - *$P(a) \rightarrow Q(a)$  and  $Q(a) \rightarrow R(a)$ , apply syllogism to get  $P(a) \rightarrow R(a)$*